Estimating Waves of Temporal Heterogeneity: A Time-Varying Parameter Model Approach*

Kenya Amano[†] Jouchi Nakajima[‡]

Abstract

Recent methodological developments in changepoint models have successfully identified structural changes in time-series cross-sectional data analysis. However, these models ignore gradual changes that happen over prolonged periods. Some scholars have developed theories to explain these slow-moving political relationships, but there are few statistical tools to empirically test these theories. To help researchers better analyze gradual changes, we propose the use of a Bayesian methodological strategy for time-varying parameter models to identify slow-moving structural changes. Specifically, we develop a time-varying parameter probit (TVPP) model, which estimates a time-varying relationship between a binary response and explanatory variables. We illustrate the utility of the TVPP models using simulated data and examples drawn from two important debates in democratization studies: (i) the identification of shifting relationships between oil wealth and democratization and (ii) the effects of income on democratic transition and consolidation. In both applications, we find that the proposed method successfully identifies substantively meaningful slow-evolving heterogeneity over sample periods.

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[†]**Corresponding Author:** Postdoctoral Fellow of Weatherhead Center for International Affairs at Harvard University (`kenya.amano@gmail.com`).

[‡]Institute of Economic Research at Hitotsubashi University

Introduction

Political scientists analyze statistical relationships from time series data to test their theories. Time series analysis faces methodological challenges if there are hidden structural changes in the data-generating process. A simple econometric model that assumes the effect on a coefficient is constant over time results in model misspecification and omitted variable bias when the data have temporal heterogeneity. Recent developments in changepoint models have proposed methods to identify these hidden breaks (Western and Kleykamp 2004; Spirling 2007; Park 2011, 2012; Hermansen, Knutsen, and Nyg 2021; Kent, Wilson, and Cranmer 2022; Park and Yamauchi 2023). The models have also been widely used in applied work in political science such as the study of courts (Hendershot et al. 2013; Pang et al. 2012), Congress Smith et al. (1999); Wawro and Katznelson (2014), civil war and battle death (Cederman, Gleditsch, and Wucherpfennig 2017; Cunen, Hjort, and Nyg 2020), terrorist attacks (Brandt and Sandler 2010; Santifort, Sandler, and Brandt 2013) and democratization and democratic consolidation (Hermansen, Knutsen, and Nyg 2021; Svolik 2015).

However, these models ignore gradual structural changes that happen over prolonged periods. If the causal relationship between x and y shifts immediately after a breakpoint, changepoint models may be appropriate to estimate the structural change. But political relations could be sticky and slow-moving. Theoretically, in the literature on institutional changes—- historical institutionalism in particular—, scholars have reexamined the assumption of *punctuated equilibrium* that causal processes and institutional changes unfold very rapidly, instead of considering the possibility that these changes can occur slowly and over time (Abbott 2001; Mahoney 2000; Mahoney and Thelen 2009; Thelen 2004; Pierson 2004; Sewell 2005; Tilly 1995; Gerschewski 2021). When the true relation slowly shifts from State A to State B, changepoint models misspecify the breakpoint as a one-time equilibrium shift. For instance, assume the demise of the Soviet Union and the end of the Cold War is a *critical junctures* (e.g. Fukuyama 1992); the changepoint model is forced to assume that many things in social relations would all change within just one year before and after 1991 because most studies of comparative politics and international relations use country-year as a unit of analysis. However, political actors and institutions exhibit path dependence, which can slow down the pace of change even when there is a clear exogenous event (Pierson 2004; Mahoney 2000; Mahoney and Thelen 2009). If the true political relationships change gradually, the abrupt shift assumption may cause biased results.

In this paper, we propose the use of time-varying parameter models to identify gradual structural changes. Based on previous research that uses a nonlinear state space model (Frühwirth-Schnatter and Wagner 2006; Park 2010; Martin and Quinn 2002), we develop Time-Varying-Parameter Probit (TVPP) model that enables us to capture incremental changes in the underlying structural relationship between a binary outcome and explanatory variables. We also extend the TVPP model to the dynamic probit model, which is widely used in democratization studies, to estimate the dynamic process of the binary outcome variable by including the lagged outcome variable. The TVPP model assumes that the time-varying parameters follow random-walk processes in the data-generating process, while the changepoint model assumes discrete changes between a finite number of hidden regimes. To estimate the latent random-walk processes within time-series data, we employ Kalman filtering and simulation smoother, which we embed within Markov chain Monte Carlo (MCMC) estimation of our model, following the Bayesian approaches proposed by the previous studies (Frühwirth-Schnatter and Wagner 2006; Carter and Kohn 1996; de Jong and Shephard 1995; Martin and Quinn 2002).

This framework has several benefits. First, it allows researchers to estimate the changes without assuming the number of breaks, which is required for the changepoint models. Second, the TVPP models are more robust to model misspecification and attenuation bias than changepoint models when the relationship between outcome and explanatory variables gradually changes over time. We show in our simulation studies that the attenuation bias in the changepoint model could produce underestimates of the coefficient shift or even false positives. Lastly, this estimation framework also allows us to answer substantively interesting questions through counterfactual simulations. In one application, we show how the trajectory of the quantities of interests would have changed when the time-varying parameter would have been fixed at a certain level compared to what actually happened.

After evaluating the performance of the TVPP models relative to changepoint models with simulated data, we illustrate the proposed methodology by applying it to two studies in democratic transition and consolidation. As Huntington (1991) describes, the data generation process of causes of democratization changes across and within the *waves* of democracy, and thus the political relations to democratic transition and consolidation are long-term, slow-moving phenomena. We first reinvestigate the proposition by Ross (2012) that oil income had a more pronounced negative effect on democratization after the 1970s. Consistent with Andersen and Ross (2014) and Ross (2012), the TVPP model finds empirical evidence that the oil's nondemocratic effects gradually emerged during the 1980s. Next, revisiting a controversy on the relationship between economic development and democratization, we examine the temporal heterogeneity in the relationship by employing the dynamic probit version of the TVPP models. We find the positive and statistically significant effect of income on democracy has gradually decreased since the beginning of the twenty century, and the effect becomes statistically insignificant after the demise of the Soviet Union and the end of the Cold War.

Finally, we see our contribution as two-fold. First, we propose to employ time-varying parameter models to identify gradual structural changes by developing the method for binary data. In political science, Beck (1983) first introduced the idea of time-varying parameter models to identify structural breaks. Martin and Quinn (2002) estimate the time-varying location of the median justice in the U.S. Supreme Court using dynamic ideal point estimation. Park (2010) also proposes a time-varying parameter model in count data. However, political scientists have underestimated the time-varying parameter approach in empirical applications compared to changepoint models. This is to our knowledge the first to demonstrate that the time-varying parameter approach has advantages in diagnosing the slow-moving changes in political relationships, which the studies of historical institutionalism value. Second, the two applications identify temporal heterogeneity in the relations to democratic transition and consolidation. In particular, the magnitude of the relationship between income and democracy gradually wanes over time, showing the gradual structural changes that occurred during the first and second reversals of democratization. More strikingly, the relationship becomes statistically insignificant after the demise of the Soviet Union and the end of the Cold War. The empirical results partly support the theories in the previous democratization studies, in which change in the international system influences the democracy-income relationship. We also demonstrate the counterfactual analysis using the time-varying parameters that may generate hypotheses, forming a basis for future research.

Empirical Strategies: Time-Varying Parameter Approach

Our time-varying parameter modeling methodology draws on a state space model developed by de Jong and Shephard (1995) and West and Harrison (1997) and later extended to nonlinear models, such as the Poisson process (Frühwirth-Schnatter and Wagner 2006). In political science, Park (2010) and Park (2011) introduce the changepoint models using this technique for binary and ordinal response data, while Martin and Quinn (2002) conduct the time-varying parameter for binary data in the context of ideal point estimation. Based on these previous studies, we extend the simple probit and the dynamic probit model with the time parameters evolving over time.

Time-Varying Parameter Probit Model

Consider a standard probit model in a panel-data form:

$$y_{it} = \begin{cases} 1 & (\text{if } z_{it} > 0) \\ 0 & (\text{if } z_{it} \leq 0), \end{cases} \tag{1}$$

$$z_{it} = \boldsymbol{x}_{it}'\boldsymbol{\beta} + \mu_i + e_{it}, \quad e_{it} \sim N(0,1). \tag{2}$$

where y_{it} is the binary response of individual (country) i at time t, for $i = 1, \ldots, I$, and $t = 1, \ldots, T$, $\boldsymbol{x}_{it}$ is a $k \times 1$ vector of explanatory variables, $\boldsymbol{\beta}$ is a $k \times 1$ vector of coefficients, and μ_i is an individual fixed effect.

We extend the probit model with time-varying coefficients:

$$y_{it} = \begin{cases} 1 & (\text{if } z_{it} > 0) \\ 0 & (\text{if } z_{it} \leq 0), \end{cases} \tag{3}$$

$$z_{it} = \boldsymbol{x}_{it}'\boldsymbol{\beta}_t + \mu_i + e_{it}, \quad e_{it} \sim N(0,1), \tag{4}$$

$$\boldsymbol{\beta}_{t+1} = \boldsymbol{\beta}_t + \boldsymbol{v}_t, \quad \boldsymbol{v}_t \sim N(0, \boldsymbol{\Sigma}), \tag{5}$$

where $\boldsymbol{\beta}_t = (\beta_{1t}, \ldots, \beta_{kt})'$ is a $k \times 1$ vector of *time-varying coefficients*. Each of the time-varying coefficients, β_{it}, follows a first-order random-walk process with the covariance matrix in equation (5) assumed to be diagonal such that $\boldsymbol{\Sigma} = \text{diag}(\sigma_1^2, \ldots, \sigma_k^2)$.[1] We assume that $\boldsymbol{v}_t$ and e_{it} are mutually independent.

We model the TVPP with unit fixed effects to control for unit-specific factors, as for-

[1] The random-walk process allows both temporary and permanent shifts in the coefficients. The drifting coefficient is meant to capture a possible non-linearity, such as a gradual change or an abrupt structural break. In practice, this assumption implies a possibility that the time-varying coefficients capture not only the true movement but also some spurious movements because the coefficient can freely move under the random-walk assumption under the risk of overfitting. In other words, there is a risk for the time-varying coefficients to overfit the data if the relations of y_{it} and x_{it} are obscure. If one assumes the time variation of the relationship between the response and explanatory variables stationary, an alternative approach is to make the time-varying coefficient follow a stationary autoregressive process such as AR(1) model.

mulated by the fixed effect parameter μ_i in equation (4). Because studies in comparative politics and international relations frequently use country-year time-series cross-section data that face omitted variable bias, it is important to remove time-invariant heterogeneity within units (countries).

Extension to Dynamic Probit Model

We extend the TVPP model to the dynamic probit model. The dynamic probit model is widely used in political science, in particular, studies of democratization as Przeworski et al. (2000) first employ this model to estimate the effect of income on democratization (Boix and Stokes 2003; Dunning 2008; Houle 2009; Przeworski et al. 2000). The model allows us to estimate the asymmetric impacts of income on the shift from an authoritarian regime to democracy and from democracy to authoritarianism in one regression. In the democratization literature, for instance, the dynamic probit model uses the dichotomous measure of democracy as the dependent variable ($y_t = 1$ if the country is democracy at the time t), and interacts with the lagged dependent variable (y_{t-1}), political regime in the previous period, with each of independent variables. Using the estimates of each independent variable and its interaction variable with the lagged dependent variable, one can interpret the coefficients on each independent variable as their probability of democratization, which is from autocracy ($y_{t-1} = 0$) to democracy ($y_t = 1$), whereas the sum of the coefficients on each independent variable and the interaction variables reflect their probability of democratic consolidation, which is from democracy ($y_{t-1} = 1$) to democracy ($y_t = 1$).

Specifically, in the TVPP model above, we replace equation (4) by the dynamic probit representation as

$$z_{it} = \boldsymbol{x}'_{it}\boldsymbol{\beta}_t + (y_{i,t-1}\boldsymbol{x}_{it})'\boldsymbol{\gamma}_t + \mu_i + e_{it}, \tag{6}$$

where $\boldsymbol{\beta}_t$ is the vector of coefficients associated with the transition from autocracy $(t-1)$ to democracy (t), while $\boldsymbol{\gamma}_t$ is the $k \times 1$ vector of coefficients associated with the state holding democracy between $t-1$ and t. Note that, when $\boldsymbol{y}_{i,t-1} = 1$, the right-hand side of the equation reduces to $\boldsymbol{x}'_{it}(\boldsymbol{\beta}_t+\boldsymbol{\gamma}_t)+\mu_i+e_{it}$, where $\boldsymbol{\beta}_t+\gamma_t$ measures the impact of the explanatory variables on the probability of democratic consolidation.

Bayesian estimation

To estimate the TVPP model, we employ a Bayesian estimation strategy. The likelihood of the model defined by equations (3)–(5) includes so many latent variables $(\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_T)$ that implementing a standard maximum likelihood estimation is computationally challenging. To overcome it, we take an approach of Bayesian inference and utilize the MCMC method in our analysis of the TVPP model. The Bayesian estimation for the standard Probit model with a Gibbs sampler has been well established (see, e.g., Chib and Greenberg 1996; Koop 2003). We extend the sampler to the one that explores a posterior distribution of the TVPP model. A key aspect of the sampling method is that the equations form a linear and Gaussian state-space model, conditional on z_{it}. de Jong and Shephard (1995) develop an efficient MCMC sampler for the linear and Gaussian state space model, which we employ in our estimation method. Their efficient sampler generates a sample from the joint posterior distribution of $(\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_T)$. With this sampler, the MCMC converges more quickly than the one that uses posterior sampling from each posterior distribution of $\boldsymbol{\beta}_t$ given $(\beta_{t-1}, \beta_{t+1})$ and other parameters recursively for $t = 1, \ldots, T$.

Let $\boldsymbol{y}$ be all the responses of $\{y_{it}\}$, for $i = 1, \ldots, I$, and $t = 1, \ldots, T$. Note that the panel data $\boldsymbol{y}$ do not need to be balanced. A modification of the sampler for unbalanced-form of panel data is straightforward. Define $\boldsymbol{\beta} = (\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_T)$, $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_I)$, and $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_k)$. Further define $\boldsymbol{z}$ as all the collections of $\{z_{it}\}$.

Setting priors $\pi(\boldsymbol{\mu})$ and $\pi(\boldsymbol{\sigma})$ for the parameters $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$, respectively, we obtain the full joint posterior distribution of the TVPP model conditional on data $\boldsymbol{y}$ given by

$$\pi(\boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\sigma}, \boldsymbol{z}|\boldsymbol{y}) \propto \pi(\boldsymbol{\theta}) \cdot \prod_{i,t} f(y_{it}|z_{it})\pi(z_{it}|\boldsymbol{\beta}, \boldsymbol{\mu}) \cdot \prod_{t=1}^{T-1} \pi(\boldsymbol{\beta}_{t+1}|\boldsymbol{\beta}_t, \boldsymbol{\sigma}). \tag{7}$$

We develop the MCMC algorithm for generating samples from the full posterior distribution. Specifically, we propose the following posterior sampler:

MCMC algorithm for the TVPP model

1. Sample $\boldsymbol{z} \,|\, \boldsymbol{y}, \boldsymbol{\beta}, \boldsymbol{\mu}$
2. Sample $\boldsymbol{\beta} \,|\, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{\mu}, \boldsymbol{\sigma}$
3. Sample $\boldsymbol{\mu} \,|\, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{\beta}$
4. Sample $\boldsymbol{\sigma} \,|\, \boldsymbol{\beta}$

The detail of each sampling step is described in Appendix A.

Finally, one reason to prefer Bayesian estimation for this model is to include unit fixed effects without concerning a bias caused by fixed effects. Generally, we might have a biased estimate when we estimate a binary response model such as the probit- and a logit-type panel data analysis with fixed effects. It is also well known that the dynamic binary model with fixed effects causes incidental parameter problems (Neyman and Scott 1948; Lancaster 2000). Suppose that the number of time periods, T, is fixed, then an estimate of the coefficient, β, is severely biased because the number of nuisance parameters grows quickly as the number of unit fixed effects increases. However, we can avoid this problem in the Bayesian approach as suggested by previous studies (e.g., Lee 2016), following the idea of conditional logit approach by Chamberlain (1980). The Bayesian approach constructs a sequence of samplers to assess a joint posterior distribution as a whole, where each sampler generates a sample from the posterior distribution conditional on other parameters in the model, as in the

MCMC algorithm explained above. It means that a part of the samplers is based on the conditional logit model, which avoids the problem as Chamberlain (1980) proposed.

Simulation Study

In this section, we will evaluate our estimation strategy using simulated data. We generate binary time series data for time length $T = 50$ with three scenarios for structural breaks. The time series data include the $k = 2$ explanatory variables in $\boldsymbol{x}_{it}$ and the time-varying parameters, (β_{1t}, β_{2t}). The first scenario (a) is the *Punctuated Equilibrium* case, in which the parameter abruptly shifts from one level (State A) to the other (State B) at a certain breakpoint. We specify the change point at $t = 26$, in the middle of the $T = 50$ sample period from no effect (($\beta_{1,1} = 0$) to 10 for β_{1t}, and 10 to no effect for β_{2t}. Second, we generate data with a gradual structural change, which we label scenario (b). The parameter in this scenario starts increasing from zero for β_{1t} (decreasing from 10 for β_{2t}) in the early period of the sample at $t = 11$, and continues to increase (decrease) until the end of the estimation period. Lastly, we consider the gradual shift reaches a new equilibrium in scenario (c). The parameter in this scenario starts and changes the same as scenario (b), but it ends its evolution in the later period at $t = 40$ until the end of the estimation period. We also examine different sizes of units in the panel data. As the number of units grows, generally, we have more information on the time-varying parameters at each time point. Specifically, we generate data with the number of units, $I = (20, 40, \ldots, 120)$. Note that we include the unit effect in this simulation as we have modeled above, which is generated from the uniform distribution $U[-1, 1]$.

We estimate the TVPP and changepoint models for these three scenarios to compare their performances. As for the changepoint model, we employ the model and the MCMC algorithm developed by Park (2010), where the coefficients shift from the regime $s = 1$ to $s = 2$ with the probability p. In the MCMC estimation for both models, we draw 5,000

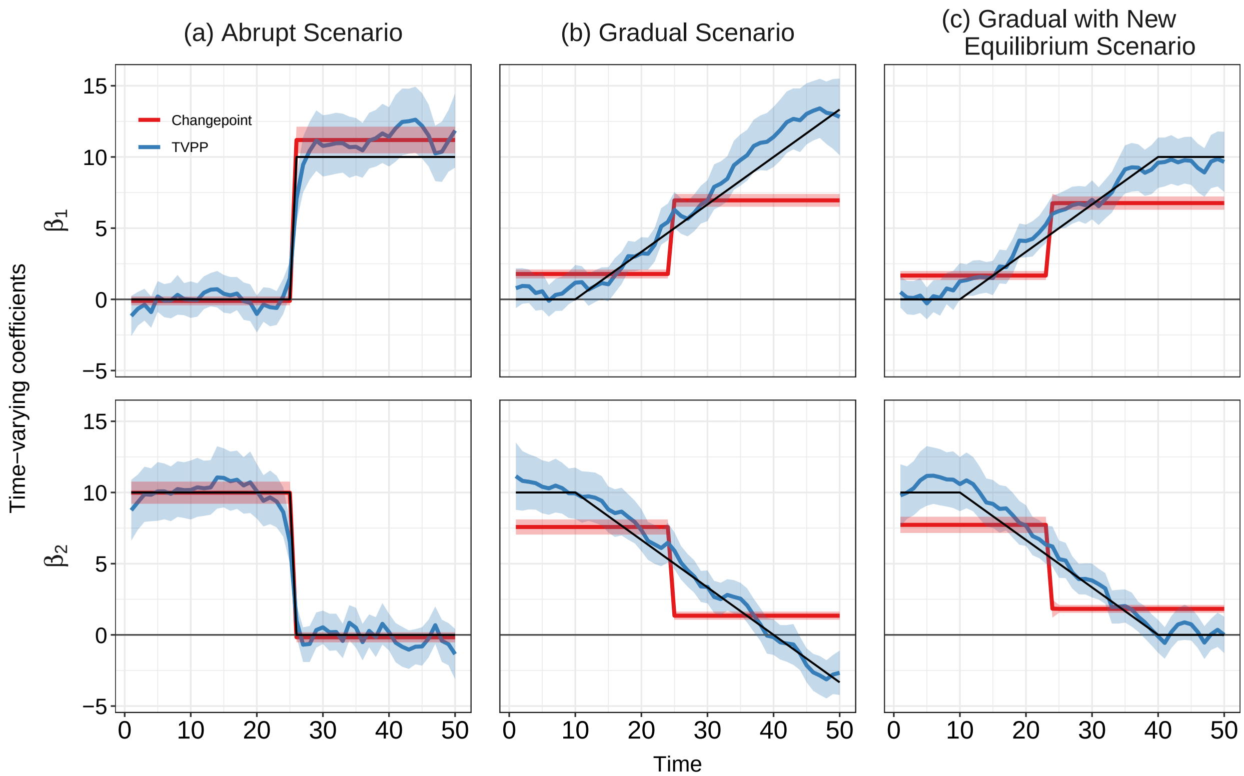

Figure 1: **Simulation Outcomes for the TVPP and the Changepoint Models.**
Note: Simulation outcomes from 50 sets of simulated data. The black, blue, and red lines are true values, the posterior means of the TVPP model, and those of the changepoint model, respectively. Shaded areas indicate 95% confidence intervals.

Table 1: **Average of the First and Last 10 Periods of Estimated Coefficients ($\boldsymbol{\beta}_{2t}$).**

			First 10 period ($t = 1, \ldots, 10$)	Last 10 period ($t = 41, \ldots, 50$)	Diff. between First and Last
(a)	**Abrupt scenario**	**Truth**	10	0	-10
		CP	9.98*	-0.2	-10.2^*
			[9.2 – 10.8]	[-0.5 – 0.2]	[-10.6 – -9.7]
		TVPP	9.84*	-0.6	-10.5^*
			[7.9 – 11.8]	[-2.0 – 0.8]	[-11.0 – -9.9]
(c)	**Gradual with new equilibrium scenario**	**Truth**	10	0	-10
		CP	7.7*	1.8†	-5.9^*
			[7.2 – 8.3]	[1.5 – 2.1]	[-6.2 – -5.6]
		TVPP	10.7*	0.2	-10.5^*
			[8.7 – 12.7]	[-1.0 – 1.4]	[-11.3 – -9.7]

Note: The table summarizes the model performances for the TVPP and changepoint models by calculating the averages of the first and last ten periods of the estimated coefficients, based on $\boldsymbol{\beta}_{2t}$ for scenario (a) and (c) in Figure 1.
* and † indicate significance at the 5% level and false positive, respectively.

samples after the initial 500 samples are discarded as a burn-in period. With this iteration size, we found that the MCMC sequence converged well. We set up the following priors: for the TVPP model, $\mu_i \sim N(0,1)$, $\sigma_j^2 \sim IG(25,25)$, for $j = 1,2$, and $\boldsymbol{\beta}_{1t} \sim N(\mathbf{0}, 100\boldsymbol{I})$, where IG denotes the inverse gamma distribution; and for the changepoint model, $\boldsymbol{\beta}_s \sim N(\mathbf{0}, 100\boldsymbol{I})$, for $s = 1,2$, and $p \sim B(10,1)$.

Figure 1 illustrates the estimated parameters from the TVPP (blue line) and the changepoint model (red line) against the true value (black line). Note that we obtain them using one dataset simulated each from (a) abrupt scenario, (b) gradual scenario, and (c) gradual with new equilibrium scenario with 100 units ($I = 100$). The solid and shaded areas indicate the posterior means and 95% credible intervals, respectively.

For Scenario (a), it is evident that the estimate of the changepoint model largely traces the true values. In particular, β_{2t} in the bottom-left panel for the changepoint model traces almost exactly the true values. The TVPP model also works well as it captures the regime shift, although the estimate fluctuates to some extent. For Scenario (b), the TVPP largely follows the true value, while the changepoint model works poorly. The changepoint model yields the regime shift as it occurs around $t = 25$ with the estimate of the pre-break and post-break regime apart from the true value. Despite the fact that the true value in the beginning (end) of the estimated period is zero (negative) for β_{1t} (β_{2t}), the changepoint model indicates the false positive relationship. In contrast, the TVPP model traces the true values well, and its estimate is statistically insignificant at the beginning of the estimated period for β_{1t}. Likewise, β_{2t} is negative and statistically significant at the end of the period. The attenuation bias produced by the changepoint model does not change even if the true value reaches a new regime in Scenario (c). In the same way, the TVPP model works better than the changepoint model in Scenario (c). Because the TVPP model can capture both the incremental slope shift and regime change, the TVPP model can trace the true values better than the changepoint model, which assumes a discrete regime change.

We also compare the magnitudes of structural changes between Scenario (a) and (c), by computing the average of the posterior means of $\boldsymbol{\beta}_{2t}$ for the first and last 10 periods of the sample (first: $t = 1, \ldots, 10$; last:$t = 41, \ldots, 50$). The true value starts from 10 and decreases to zero. The rightmost column of Table 1 indicates the difference in the average of the posterior means between the first and last periods. In (a) an abrupt scenario, the changepoint model estimates the difference that is about the same as the truth value, while the TVPP model has a slightly larger impact and a wider credible interval. On the other hand, in (c) a gradual with a new equilibrium, the changepoint model estimates the smaller magnitude of the structural change (-5.9) than the true value (-10) and the TVPP model (-10.5) because the changepoint includes transitional periods as the post-break effect. Moreover, the average of the last 10 periods in the changepoint model indicates a positive and statistically significant result (1.8), causing a false positive indicated as † in Table 1[2].

To further evaluate the performance of each model, we compute the root mean squared estimation error (RMSE) of the posterior draws. Let β_{it} denote the true value and $\beta_{it}^{(g)}$ denote samples generated at g-th iteration of the MCMC. Following Park (2011, 2012), we compute the RMSE as follows:

$$\text{RMSE}_{\boldsymbol{\beta}} = \sqrt{\frac{1}{G}\sum_{g=1}^{G}\left\{\frac{1}{(t_1 - t_0 + 1)p}\sum_{t=t_0}^{t_1}\sum_{i=1}^{p}\left(\beta_{it}^{(g)} - \beta_{it}\right)^2\right\}}, \tag{8}$$

where G is the iteration size of the MCMC samplers. We set the evaluation period as the one in which we specify the gradual change, $t_0 = 15$ and $t_1 = 35$. We repeat the estimation for 50 sets of data, where we compute this RMSE for each simulation and average the RMSE across 50 sets.

Figure 2 shows the RMSE in the changepoint model (red line) and the TVPP model (blue line) for each simulation with different sizes of units as indicated by the horizontal

[2] We also extend the period after the structural change to see how the length of the post-break period contributes to the changepoint model. The false positive of the changepoint model is not resolved even if we extend the estimation period to 100 from 50. See more details in Appendix B.

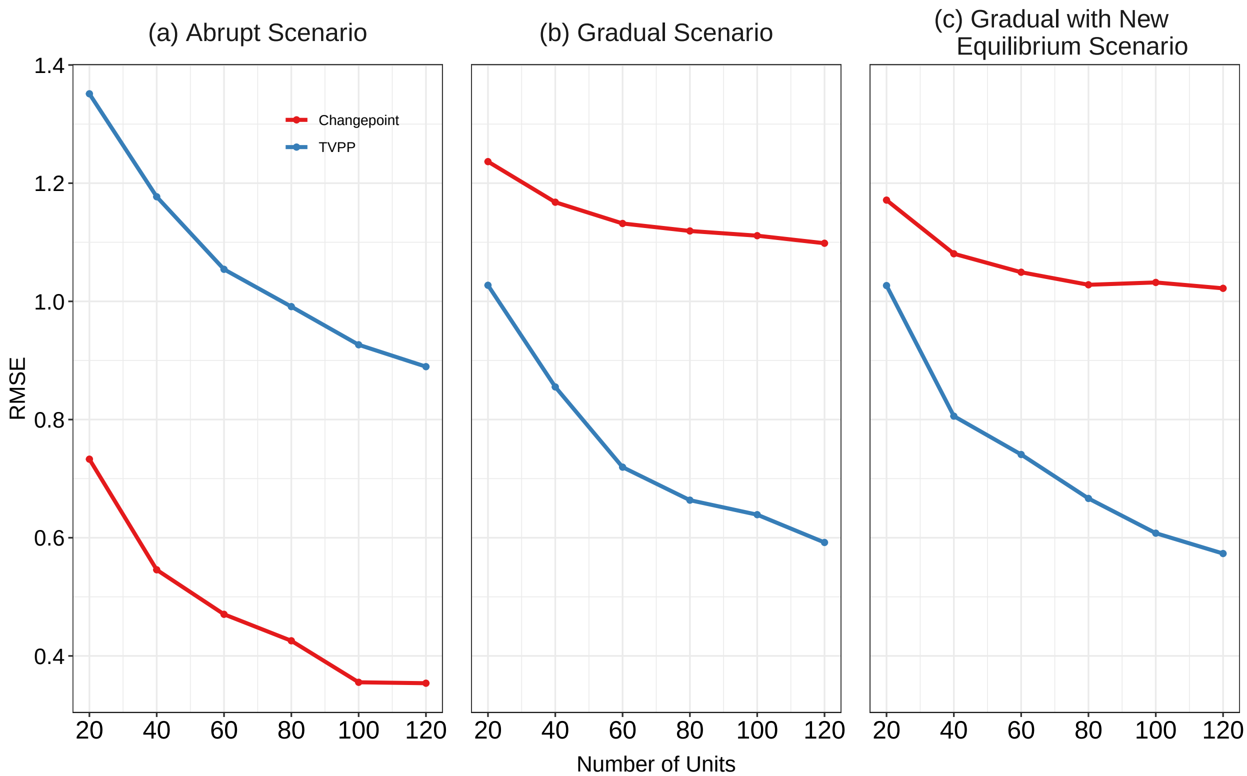

Figure 2: **Root Mean Squared Error of the TVPP Model and the Changepoint Model from Simulated Data.**
Note: Simulation outcomes from 50 sets of simulated data. The true values of each scenario are followed by Figure 1. A lower RMSE indicates good predictive accuracy.

axis. As expected, the changepoint model has smaller RMSEs than the TVPP model in (a) abrupt change scenario, but the TVPP model outperforms the changepoint model in (b) gradual change scenario and (c) gradual with new equilibrium scenario. Moreover, while the estimation errors accumulated in the changepoint model as the number of units increases, the TVPP sharply decreases the RMSE when the number of units increases. This result confirms that our TVPP model performs practically well, capturing both the gradual and abrupt changes in the data-generating processes.

Application 1: Democracy and Natural Resource

Our first application is the political relationship between resource wealth and democracy. Besides income, resource wealth, especially petroleum, is considered the primary factor of regime stability. The debate over the political resource curse, the claim that higher levels of oil wealth make autocratic governments more stable and hence less likely to transition to democracy, has drawn attention to democratization studies. After the extant literature advanced and debated theories, data, and empirical methods, many studies are broadly consistent with the claim that oil wealth makes autocratic governments more stable but with certain conditions (Ross 2012; Andersen and Ross 2014; Tsui 2011; Ahmadov 2014).

While much of this research has tried to clarify the conditions under which petroleum wealth has negative impacts on democracy, one important condition is a temporal dimension. In their critique of the seminal work by Haber and Menaldo (2011) that dismisses the resource curse argument, Andersen and Ross (2014) argue that there was a structural break in the relationship between oil wealth and democracy. The *big oil change* they call occurred in the late 1970s when the oil industry was transformed by a wave of nationalizations and contract revisions that enabled the governments of host countries to seize control of these rents. To identify this possible structural break, Andersen and Ross (2014) estimate the term interacting oil wealth variables with time dummies, and Ross (2012) subsets the sample data into before and after 1980 to estimate the relationship between the transition to democracy and oil income. With these strategies, these studies conclude that structural change causes the resource curse, while there might be no resource curse before the 1970s.

Yet, the estimations with time dummies or subsampling produce estimation biases because the results can vary depending on which period is specified. Even using strong prior knowledge about the timing, the number, and the duration of structural changes, researchers cannot statistically validate their prior if the change is incremental. The changepoint model may find the structural change of oil wealth effect, but the abrupt structural change assumption is not suitable in this case because the *big oil change* theory claims that the structure of

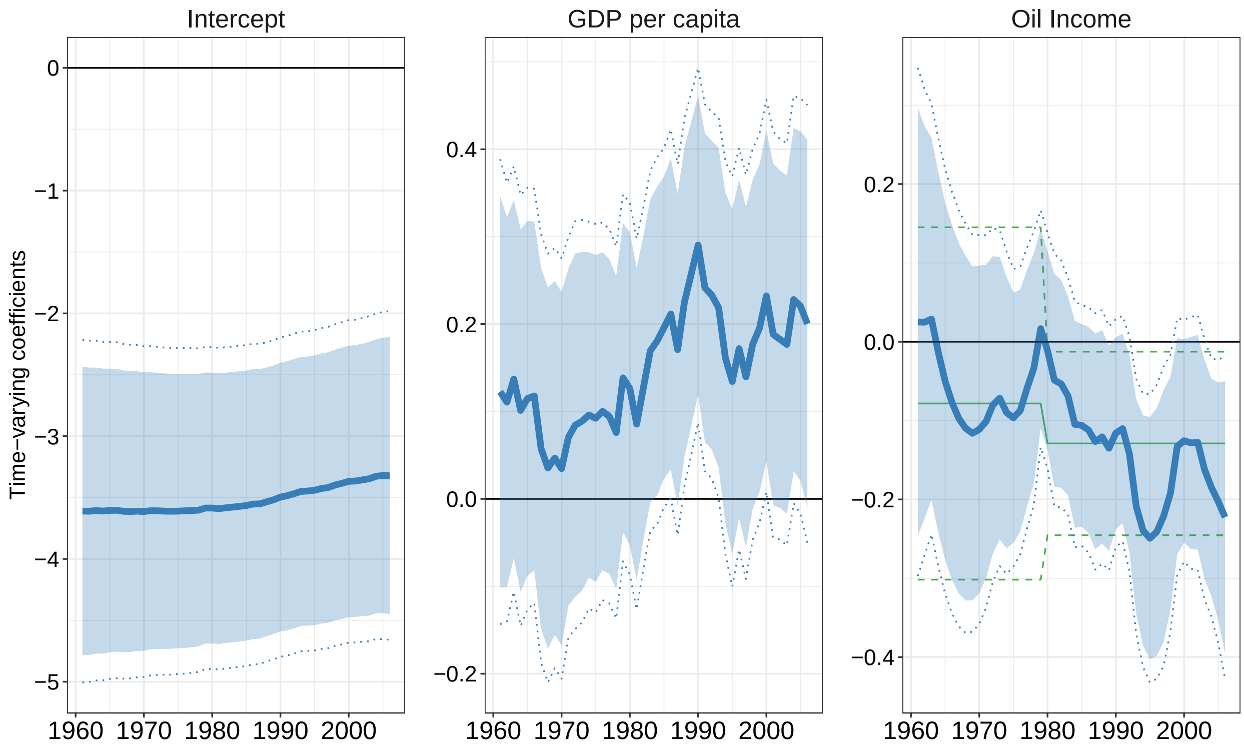

Figure 3: **Time-Varying Parameters for the Oil Effect on Democratic Transition.** Note: The outcome variable of this model is democratic transition, taking value 1 if a country has regime transition at year t. The blue line is the posterior means of the TVPP model, and the green solid line is the coefficient estimated by Ross (2012). The shaded areas and the dotted lines are 90% and 95% confidence intervals, respectively.

the international oil industry was incrementally changed. Andersen and Ross (2014) describe that "(t)he transfer of rents often took place over a 5- or 10-year period, as governments **gradually** gained control over foreign assets, renegotiated or abrogated contracts, reorganized existing national oil companies or established new ones, and developed new regulations. This makes it hard to identify a single year when the salient dimensions of nationalization took place". Therefore, the TVPP model is well suited to capture the gradual change of the oil effect on democracy.

In this application, we reanalyze Ross (2012), which estimates the effect of the total oil income on binary data of regime transition covering 174 countries from 1960 to 2006. To test the temporal effect, the study subsamples the data into two periods: 1960-1979 and 1980-2006. However, dividing the data by 1980 does not have unequivocal reasons to be a breakpoint. Following Ross (2012), we estimate the time-varying coefficient of the effect of total oil income (log) on the timing of regime transition from autocracy to democracy, taking value one if the country i has regime transition at year t, otherwise zero. The model also includes income (log) and regime duration since 1946 as covariates with country-fixed effect.

Figure 3 plots the posterior estimates for the time-varying coefficients of total oil income in the blue line and shaded area. The right panel shows that the coefficients are between zero to -0.1 before 1980, but the 95% credible intervals (shaded area) are wide and overlap zero: the effect of total oil income is not statistically significant. After 1980, the time-varying coefficient clearly declined until the middle of the 1990s, indicating the level shift of the coefficients. While the 95% credible intervals overlap zero in the latter of the estimation periods, the 90% credible interval (dotted lines) indicates that total oil income has a negative effect and is statistically significant.

Compared to the original estimation, the timing of the structural break is similar, but the magnitude is distinct. The green horizontal lines indicate the coefficients of total oil income in the original estimations by Ross (2012), showing the estimates prior to 1980 in the original study and the TVPP model are almost equivalent. The estimated timing of the structural change in the TVPP model is also almost identical to those assumed in the original study, but the relationship between democratization and oil wealth gradually changes from 1980 through the mid-1990s. This supports the theoretical argument that the *big oil change* would take place over 10 years, which the simple logit estimation could not reveal. Moreover, the magnitude of the coefficients for the post-break is larger for the time-variant estimates, suggesting the nondemocratic oil effect is more robust than the originally estimated.

We also estimate the effect of total oil income on the binary political regime type because the time-varying parameters in Figure 3 are fluctuate due to the small number of democratization events (the proportion of $y = 1$ is 2.3 percent). In this model, the outcome variable takes the value one if the country i is democratic at year t, or zero if authoritarian. Compared to the previous model, this model has more information on the outcome variable (the

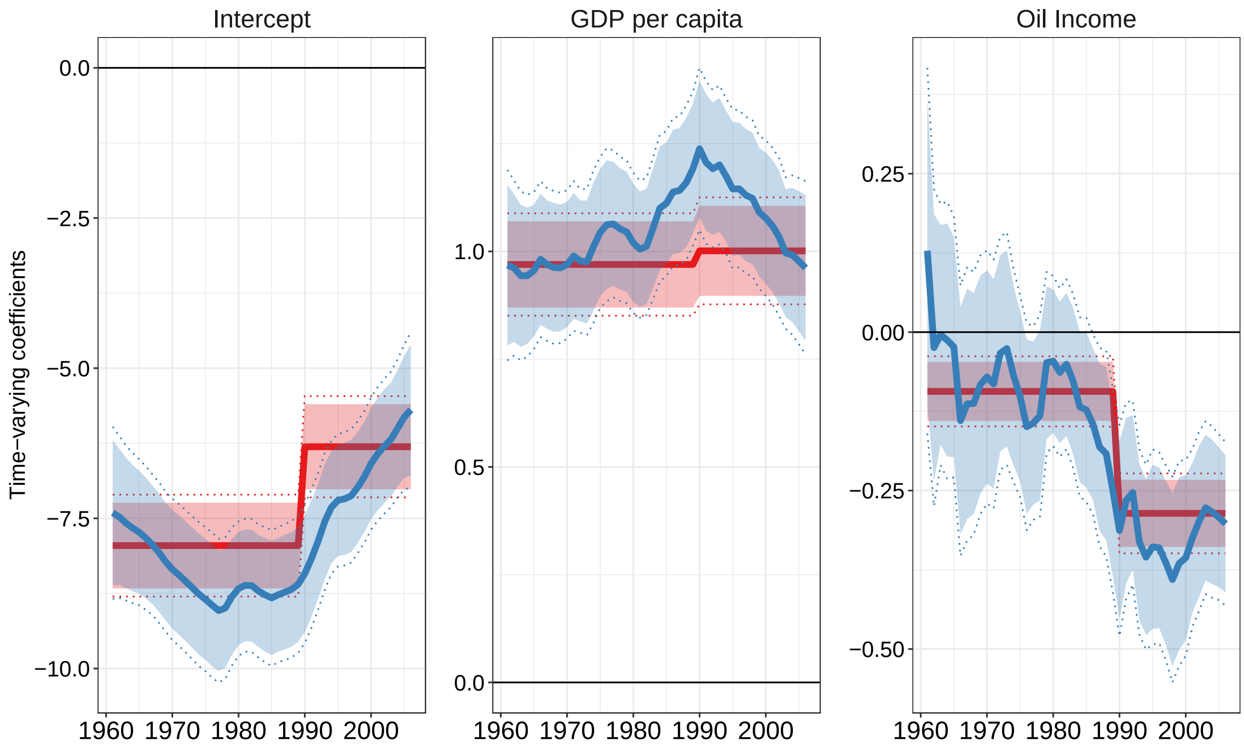

Figure 4: **Time-Varying Parameters for the Oil Effect on Binary Democracy Index.** Note: The outcome variable of this model is the level of democracy in a binary index, taking value 1 if a country is democracy at year t. The blue and red lines are the posterior means of the TVPP model and the changepoint model. The shaded areas and the dotted lines are 90% and 95% confidence intervals, respectively.

proportion of $y = 1$ is 43.8 percent), which could yield more stable time-varying coefficients [3]. Figure 4 indicates an almost parallel trend with the previous model, but the time-varying coefficients are smothered both in total oil income and GDP per capita. Moreover, the effect of total oil income declines in the middle of the 1980s, and it gets statistically significant at the 5% level. Thus, the result suggests that oil's antidemocratic effect has grown during the 1980s. Given the data limitation on pre-1960, the confidence intervals at the beginning

[3] We change the prior on σ_j^2 from the simulation study above to smooth the time-varying parameters. In the model with the outcome variable of the transition to democracy (Figure 3), we set $\sigma_j^2 \sim IG(400, 2)$, whose mean and standard deviation are roughly (0.005, 0.0003). We employ this rather tighter prior with a smaller mean than the uninformative prior used in the simulation study because the estimation with the uninformative prior yields wild fluctuations in the estimate of the time-varying parameters due to a few samples that take one in the outcome variables (the proportion of $y = 1$ is 2.3 percent). On the other hand, for the model with the outcome variable of the binary democracy index (Figure 4), in which the proportion of $y = 1$ is 43.8 percent, we set a looser prior as $\sigma_j^2 \sim IG(100, 10)$, whose mean and standard deviation are roughly (0.1, 0.01).

of the data are wider, so we cannot definitely judge the oil effect at the beginning of the estimated period.

To compare the performance of the TVPP models to the changepoint model, we also estimate the oil effect on the state of the political regime using the changepoint model used in the simulation. The red lines and shaded area in Figure 4 show the abrupt shift of the coefficient of oil income in 1990, indicating the model neglects the gradual change in the international oil industry starting in the late 1970s. Moreover, the coefficient of the pre-break periods is negative and statistically significant at the 5% level, suggesting that the oil's antidemocratic effect is always significant throughout the estimated periods. This result is contradicted by the original theoretical argument of the emergence of the resource curse after 1980. Thus, the changepoint model causes attenuation bias because the coefficient of pre-break includes the transition periods, deviating from the original theoretical argument. Moreover, the magnitude of the coefficient in the post-break period is smaller than in the TVPP model.

Meanwhile, the time-varying coefficients of GDP per capita in Figure 4 show that the effect of GDP per capita on democracy is positive and statistically significant over the estimated period. The effect increases from the beginning of the estimated period and then the effect sharply declines after 1990. We examine this democracy-GDP relation in depth in the next section.

Application 2: Democracy and Development

Our second application is the statistical relationship between democracy and economic development that has been the center of debates in comparative politics. Since Lipset (1959) developed the modernization theory, scholarship has examined the causal effect of economic development on democratization. Theoretically, the critics of the modernization theory argue that the statistical association between income and democracy holds because the survival of wealthy democracies is more likely, showing that democratization is random than systematic (Przeworski et al. 2000). Acemoglu and his colleagues (Acemoglu et al. 2008, 2009) also reject the modernization hypothesis, arguing that the historically rooted institutions affect the long-term relationship between economic development and political regime. Others, on the other hand, develop conditional modernization theory; scholars find new theories to hold economic modernization theory by adding the conditions and triggers of democratization (Boix 2011; Kennedy 2010; Miller 2012; Treisman 2020). Some others also underscore the international factors that cause the spatial and temporal clustering of democratization may change the relationship between income and democratization (Huntington 1991; Levitsky and Way 2006; Boix 2011; Cook, Hays, and Franzese 2023). Despite the developments of theories, data, and empirical methods, the literature on the causes of democracy is unsettled.

This is partly because the empirical results depend on which temporal dimension is analyzed, such as whether it includes the nineteenth-century, prewar, or early twenty-first centuries. For instance, Boix (2011) separately estimates the effect of GDP per capita on democratization in different periods, resulting in that the substantive effects are significant during the first and third wave of democratization, but there are no effects during other periods. Treisman (2020) also indicates the idiosyncratic temporal effects by conducting the time interaction model. Cook, Hays, and Franzese (2023) and Abramson and Montero (2020) develop the models to capture the lag effects of both space and time. With respect to the temporal heterogeneity approach, Hermansen, Knutsen, and Nyg (2021) employ the changepoint model that identifies the structural breaks when the Berlin Wall fell.

We employ the TVPP model to estimate the temporal effects of income, as a proxy of economic development, on democratization and democratic consolidation, revising the seminal work by Przeworski et al. (2000). We follow Przeworski et al. (2000) in estimating a dynamic probit specification, where y_{it} is the dichotomous variable capturing democracy, taking the value one if the country i is democratic at year t, or zero if authoritarian. The classification of political regimes is based on a dichotomous regime defined by Boix, Miller, and Rosato (2013). Because the original work by Przeworski et al. (2000) only includes the sample period from 1960 to 1990, we extend the timeframe further back in time, using the Boix-Miller-Rosato dichotomous coding of democracy, starting in 1800 (Boix, Miller, and Rosato 2013).

In a dynamic probit model, we regress regime type on lagged values of GDP per capita and interact with the lagged dependent variable (i.e. whether the country was a democracy in the previous period) with each of the independent variables as follows:

$$\Pr(y_{it} = 1) \quad = \phi\{\beta_{1t} + \beta_{2t} GDP_{i,t-1} + \gamma_{1t} y_{i,t-1} + \gamma_{2t} y_{i,t-1} GDP_{i,t-1} + \mu_i\} \tag{9}$$

where $\Pr(y_{it} = 1)$ signifies the probability that country i is a democracy in year t, $\phi(\Delta)$ is the cumulative distribution function of the standard normal distribution, $y_{i,t-1}$ is a lagged democracy variable, and μ_i is country fixed effects. This model estimates two β_{it} that show the relationship between democracy and GDP per capita in autocracies (the independent variables entered on their own) and two γ_{it} in democracies (the sum of the direct effect of each interaction variable with the lagged democracy variable). Thus, in the effect of income, the coefficients on GDP per capita variable, β_{2t}, reflects its association with *transition to democracy*, whereas the sum of estimates and the interaction variables, $\beta_{2t} + \gamma_{2t}$, reflects its association with *democratic consolidation*.

While Przeworski et al. (2000) does not include six oil-exporting countries (Bahrain, Kuwait, Oman, Qatar, Saudi Arabia, and the United Arab Emirates), we include those countries.

Figure 5 plots the posterior estimates for the time-varying coefficient for *Democratic transition* in the top panel and *Democratic consolidation* in the bottom panel. Looking at this estimated long-time series data in the top-right panel, while the credible intervals at the beginning of the estimated period are wider due to fewer number of countries, the coefficients of GDP per capita for *Democratic transition* increase until the mid-twenty century, and then,

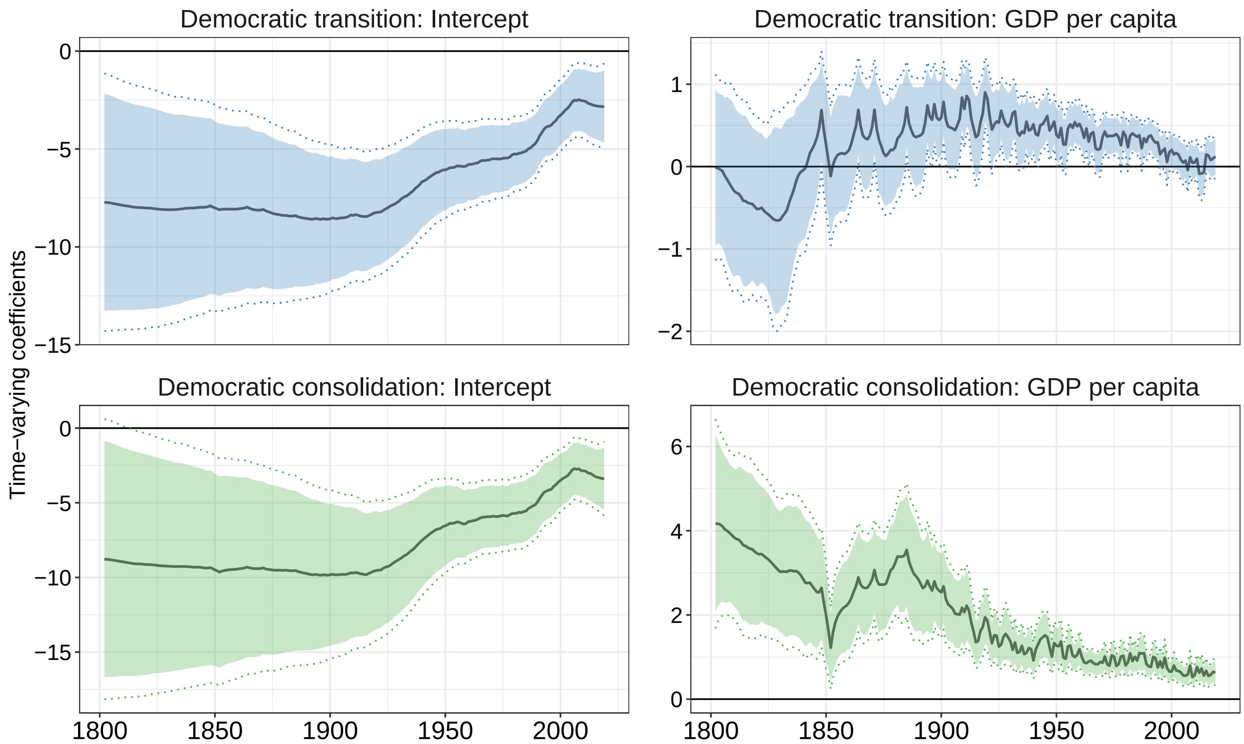

Figure 5: **Time-Varying Parameters for the Democracy-Income Relationship.** Note: The outcome variable of this model is the level of democracy in the binary index, taking value 1 if a country is democracy at year t. The blue and green lines are the posterior means of the TVPP model for democratic transition, β_{2t} in equation (9), and democratic consolidation, $\beta_{2t} + \gamma_{2t}$ in equation (9), respectively. The shaded areas and the dotted lines are 90% and 95% confidence intervals, respectively.

it gradually wanes toward the end of the data periods. On the other hand, the coefficients of GDP per capita for *Democratic consolidation* gradually decline over prolonged periods, and the sign of the coefficients are always positive and statistically significant at the 5% level.

Following Huntington's third wave classification (Huntington 1991), we zoom in the coefficients of GDP per capita for *Democratic transition* and draw the lines indicating the years of the breaks of the waves. We also add two more lines in 1991 as the end of the Cold War, and 2007 as the beginning of the Great Financial Crisis. Figure 6 indicates that after the beginning of the first reverse wave and the second reverse wave, the coefficients for *Democratic transition* show the level shift to around 0.5, and to 0.3, respectively. Then, after the demise of the Soviet Union in 1991, the coefficients clearly decline, and the credible intervals

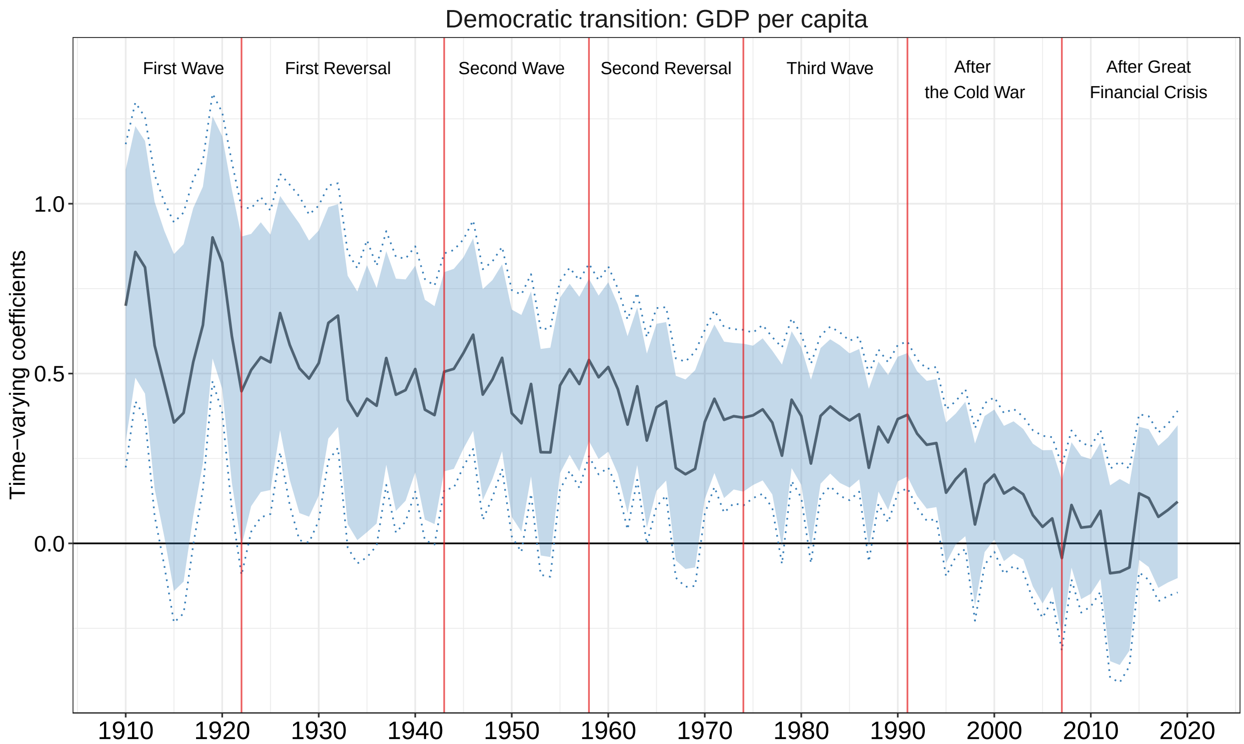

Figure 6: **Time-Varying Parameters for the Income Effect on Binary Democracy Index.**
Note: The outcome variable of this model is the level of democracy in the binary index, taking value 1 if a country is democracy at year t. The blue line is the posterior means of the TVPP model for democratic transition, β_{2t} in equation (9). The shaded areas and the dotted lines are 90% and 95% confidence intervals, respectively. The red lines indicate the end of the first wave (1922), the first reversal (1943), the second wave (1958), the second reversal (1974), the third wave (1991), and after the Cold War period (2007).

overlap zero, exerting a structural change in the relationship between economic growth and democratization from the positive income effect to statistically insignificant relations.

The estimated result of the temporal heterogeneity is distinct from the Boix (2011)'s estimation. In his estimation, Boix (2011) divides the time-series data into five periods: pre-first wave (1800–49); the first wave (1850–1920); the first reversal (1920–44); the second wave and reversal (1945–75); and the third wave (1976–2000). He finds that the positive and statistically significant relationship between income and democracy only during the first wave and third wave, while we find the statistical relation holds until the end of the Cold War. Moreover, Boix (2011)'s classification fails to specify the timing of the shift in

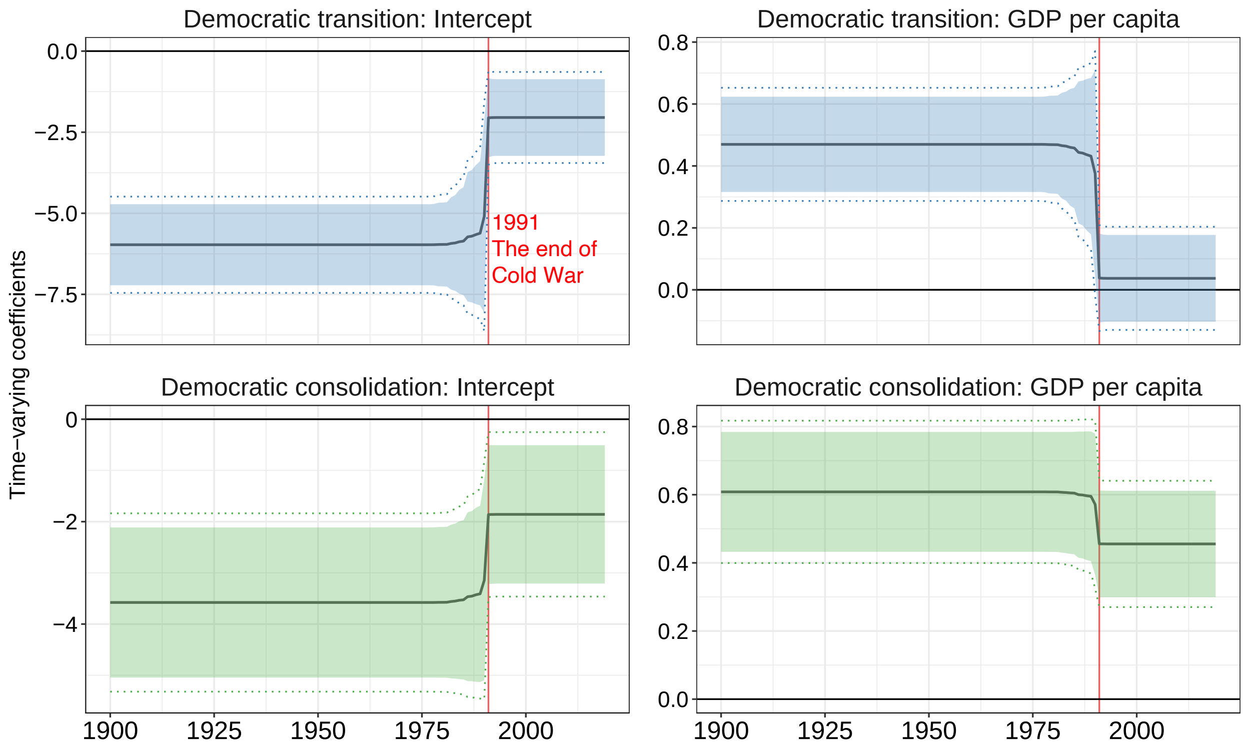

Figure 7: **Changepoint Model for the Income Effect on Binary Democracy Index.** Note: The outcome variable of this model is the level of democracy in the binary index, taking value 1 if a country is democracy at year t. The blue and green lines are the posterior means of the changepoint model for democratic transition, and democratic consolidation, respectively. The shaded areas and the dotted lines are 90% and 95% confidence intervals, respectively.

the income-democracy relation because the subsampling approach relies on the researcher's subjective decision on the possible breaks.

Compared to the changepoint model, the timing of the break is consistent with the previous study by Hermansen, Knutsen, and Nyg (2021). Employing the changepoint model to the continuous measure of democracy (V-Dem), they find the break occurring in 1989 when the Berlin Wall fell. We also estimate the breakpoints with Park's Bayesian changepoint model (Park 2010). Figure 7 shows the model identifies the break in 1991, and the coefficients become statistically insignificant after the 1991 break. However, it is difficult to capture the nuanced gradual change of the relations.

One possible interpretation for the break and the change of the relationship is the change of the international system (Boix 2011; Dunning 2004; Gleditsch and Ward 2006; Levitsky and Way 2006). After the Soviet Union disintegrated and the Cold War ended, the influence of the Soviet Union on many autocratic regimes was removed, and thus the democratic transition occurred without the democracy-income channel. On the flip side of the coin, the uncontested hegemon of the United States in the last two decades may support a robust wave of democratization (Boix 2011; Levitsky and Way 2006). Another explanation is the rise of hybrid regimes, which are categorized as authoritarian regimes in the binary classification of this model. By introducing elections, legislature, or other accountability institutions to co-opt the opposition, the hybrid regimes could maintain their regime while at the same time promoting economic growth (Wright 2008; Gandhi and Lust-Okar 2009; Shih 2020).

This interpretation is also confirmed by the time-varying parameter of the intercept. The time-variant intercepts indicate the probabilistic impact on democratization by age, excluding the income effect: the larger the coefficient, all else equal, the more likely a given age is to democratize than other ages. Thus, the top-right panel of Figure 5 shows that the slope of the time-varying coefficient becomes steeper after 1991, suggesting that a country is more likely to democratize than during the Cold War. Moreover, both the coefficients of GDP per capita and intercept suggest that the transition to democracy is less likely after 2007 because the income effect is not statistically significant and the age effect has declined.

Finally, to further evaluate the income effects, we also conduct the counterfactual analysis by using conditional posterior distributions. We construct a counterfactual scenario that the effect of GDP per capita on democratization would not be changed after the end of the Cold War. Specifically, the coefficients of GDP per capita from 1992 to 2019 are extrapolated by the mean of the time-varying coefficients from 1950 to 1991. Figure 8 shows the number and the share of democratic countries with the counterfactual result. The green lines indicate the counterfactual results, and the shaded area and the dotted lines are the 90% and 95% credible intervals. We observe that the number and the proportion of democracies increase faster in the counterfactual scenario than in the actual trajectory after 1991. Although the 95% credible intervals overlap the actual values in both plots, the predicted values after 2000

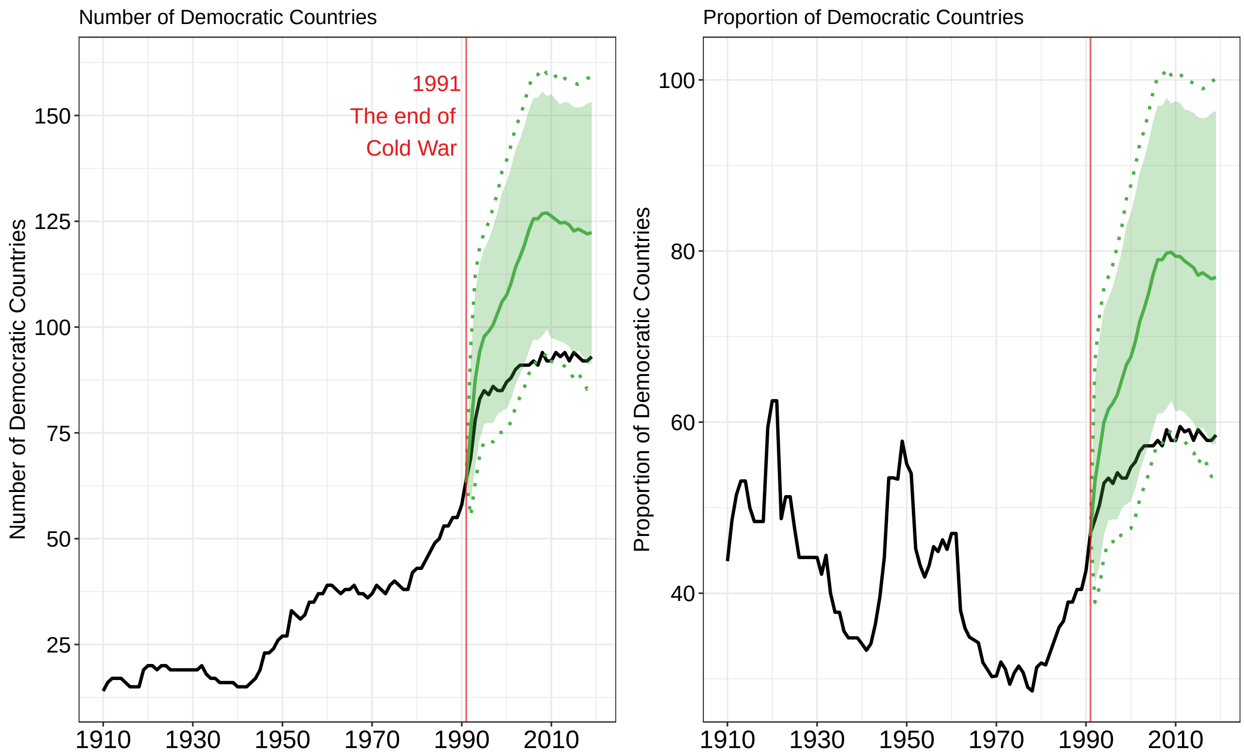

Figure 8: **Counterfactual Analysis on After the End of the Cold War.**
Note: This figure reports results from a counterfactual study with a scenario that the effect of GDP per capita on democratization would not be changed after the end of the Cold War. The green lines of the left and right panels estimate the number of democratic countries and the proportion of democratic countries. The shaded areas and the dotted lines are 90% and 95% confidence intervals, respectively.

are statistically significant at the 10% level. Thus, the number of democracies declines after 2007 because the income shock occurred during the Great Financial Crisis and the age effect estimated from the intercept negatively impacts the democratic transition and consolidation.

Conclusion

In this article, we introduced the use of the time-varying parameter model for diagnosing and modeling the gradual changes of temporal heterogeneity in politics. The findings of the simulation study and two applications suggested that the proposed methods help capture the incremental changes in historical panel data analysis, avoiding model misspecification and attenuated bias compared to the changepoint model. For example, we show in the simulation study that the changepoint model has the risk of producing false positive or false negative when the political relationships shift gradually because the assumption of abrupt, discrete level shifts in the changepoint model ignores the transition periods. In our application to the oil curse study by Ross (2012), we also demonstrate that the changepoint model causes a false negative impact of oil effect on democracy, while the TVPP model shows statistically insignificant relations before the structural change in the international oil industry, suggesting the oil curse theory is dismissed before the 1980s.

Our reanalysis of the relationship between economic development and democracy finds the temporal heterogeneity over the long historical period. The finding supports the conditional modernization theory that assumes the magnitude of the effect of economic development may be stronger or weaker in different periods. In contrast to Boix (2011)'s finding, however, we show that modernization theory holds from the nineteenth century until the end of the Cold War, and the theory does not hold, especially after the Great Financial Crisis starting in 2007. It is beyond the scope of this article to investigate the reasons that formulate this temporal heterogeneity, but our estimation results and the counterfactual analysis using the time-varying parameters advance a more nuanced understanding of the relationship between economic development and democracy.

Moreover, we believe that the utility of time-varying parameter models provides a potential avenue to investigate political relationships beyond democracy. A slow, gradual institutional change is often ignored in empirical study, so the theories in historical institutionalism have been tested mainly in qualitative analysis. We demonstrate that the time-varying parameter models could test a variety of institutional changes without having a strong prior temporal heterogeneity. Because our TVPP model is limited to analyzing binary data, future research to broadly apply time-varying parameter model will develop methods for beyond binary models, such as ordinary data and instrumental variable models.

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Appendix

A Sampling Algorithm of the TVPP Model

This appendix fully describes the sampling algorithm of the TVPP model. Each conditional sampler is standard except the Kalman filter and simulation smoother used to efficiently generate the time-varying parameters $\boldsymbol{\beta}$. We apply the algorithm of de Jong and Shephard for the generation of $\boldsymbol{\beta}$. A key aspect in this algorithm is rearranging the originally nonlinear state-space form representation of the TVPP model into a conditionally linear state-space form given the latent variable, $\boldsymbol{z}$. The detail of the sampling follows.

Sampling $\boldsymbol{z}$

We generate z_{it} from its conditional posterior distribution given other parameters. The conditional posterior distribution results in the truncated normal distribution $TN(\boldsymbol{x}_{it}'\boldsymbol{\beta}_t + \mu_i, 1)$. The range of the distribution is $z_{it} > 0$ if $y_{it} = 0$, and $z_{it} \leq 0$, otherwise. We use a rejection sampling where we draw the sample from the normal distribution and keep throwing the sample away until it falls into the corresponding range (Tierney, 1994). We generate z_{it} for $i = 1, \ldots, I$, and $t = 1, \ldots, T$, separately.

Sampling $\boldsymbol{\beta}$

We generate $\boldsymbol{\beta}$ using the well-established sampler for the state variables of the linear Gaussian state space model (de Jong and Shephard, 1995). We apply the Kalman filter and simulation smoother for the state space model conditional on $\boldsymbol{z}$ and other parameters:

$$
\begin{aligned}
\tilde{\boldsymbol{z}}_t &= \boldsymbol{x}_t'\boldsymbol{\beta}_t + \boldsymbol{e}_t, \quad \boldsymbol{e}_t \sim N(\boldsymbol{0}, \boldsymbol{I}), \quad t = 1, \ldots, T, \\
\boldsymbol{\beta}_{t+1} &= \boldsymbol{\beta}_t + \boldsymbol{v}_t, \quad \boldsymbol{v}_t \sim N(\boldsymbol{0}, \boldsymbol{\Sigma}), \quad t = 1 \ldots, T-1,
\end{aligned}
$$

where $\boldsymbol{e}_t = (e_{1t}, \ldots, e_{It})'$, $\tilde{\boldsymbol{z}}_t = (\tilde{z}_{1t}, \ldots, \tilde{z}_{It})'$, $\tilde{z}_{it} = z_{it} - \mu_i$, and we assume that $\boldsymbol{\beta}_1 \sim N(\boldsymbol{m}_0, \boldsymbol{Q}_0)$, where $\boldsymbol{Q}_0 = \mathrm{diag}(q_{01}^2, \ldots, q_{0k}^2)$. This specification is exactly the standard linear state-space model representation (de Jong and Shephard, 1995, pp. 343). We apply the algorithm of generating the state variables to our model straightforwardly.

Sampling $\boldsymbol{\mu}$

We specify the prior for $\boldsymbol{\mu}$ as $\boldsymbol{\mu} \sim N(\boldsymbol{\mu}_0, \boldsymbol{V}_0)$, where $\boldsymbol{V}_0$ is a diagonal matrix. We define a matrix of dummy variables by , where each row corresponds to the unit and time (i, t), and each column to unit effect, for $i = 1, \ldots, I$. The i-th column of takes one, corresponding to the row (i, t), and zero otherwise. Then, we obtain the conditional posterior distribution $\boldsymbol{\mu} \,|\, \cdot \sim N(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{V}})$, where

$$\hat{\boldsymbol{V}} \;=\; \left(\boldsymbol{V}_0^{-1} +'\right)^{-1}, \quad \hat{\boldsymbol{\mu}} \;=\; \hat{\boldsymbol{V}} \left(\boldsymbol{V}_0^{-1}\boldsymbol{\mu}_0 +' \hat{\boldsymbol{z}}\right),$$

and each element of $\hat{\boldsymbol{z}}$ is $z_{it} - \boldsymbol{x}_{it}'\boldsymbol{\beta}_t$.

Sampling $\boldsymbol{\sigma}$

We set the prior for σ_j^2 as $\sigma_j^2 \sim IG(n_{0j}/2, S_{0j}/2)$, where IG denotes the inverse gamma distribution. The conditional posterior distribution is given by $\sigma_j^2 \,|\, \cdot \sim IG(\hat{n}_j/2, \hat{S}_j/2)$, where

$$\hat{n}_j \;=\; n_{0j} + T - 1, \quad \hat{S}_j \;=\; S_{0j} + \sum_{t=1}^{T-1} (\beta_{j,t+1} - \beta_{jt})^2,$$

for $j = 1, \ldots, k$.

B Simulation for Changing the Post-Break Period

In the gradual with new equilibrium scenario simulation, we find the changepoint model causes the false positive when the true value moves to zero from 10 during the new equilibrium. To see if the false positive of the changepoint model is resolved, we extend the estimation period after the structural changes. We assume the changepoint model can estimate the coefficient in the new equilibrium closer to the true value when there is a longer post-break period because the inclusion of the transitional periods as the post-break becomes less impactful.

We extend the post-break period to 100 ($t = 41, \ldots, 100$) from 50 ($t = 41, \ldots, 50$), while the timing of the gradual structural change is constant ($t = 11, \ldots, 40$). Figure B1 shows that the new result is overall the same as the previous estimation. The TVPP model works better than the changepoint model in both cases, while the changepoint model estimates better in the longer post-break period. The coefficient for the new estimation indicates 0.6 (95% Credible Intervals: 0.4-0.8), while the previous estimation is 1.8 (95% CI: 1.5-2.1). Given that most studies of comparative politics and international relations use country-year as a unit of analysis and it is difficult to use the time series of more than 100 years in many cases, the changepoint model is likely to cause biased outcomes when the structural change occurs gradually.

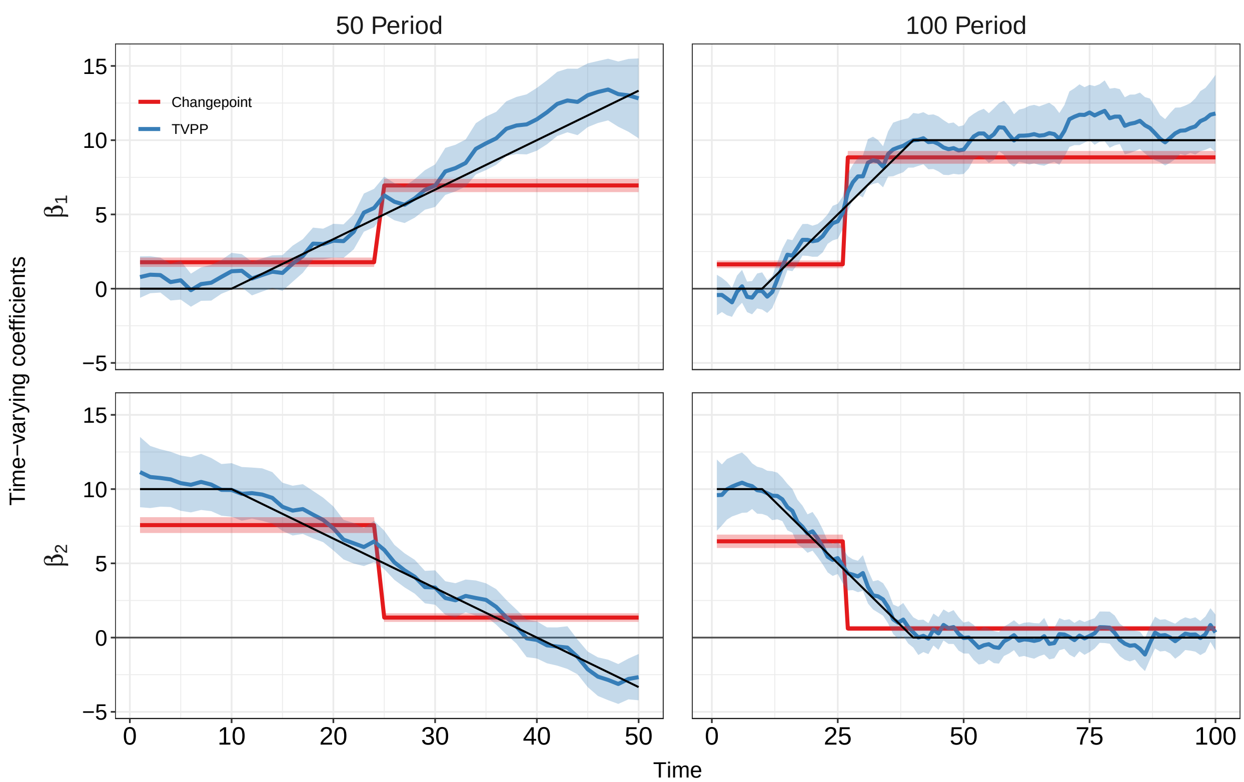

Figure B1: **Simulation for Changing the Post-Break Period.**
Note: Simulation outcomes from 50 sets of simulated data. The black, blue, and red lines are true values, the posterior means of the TVPP model, and those of the changepoint model, respectively. Shaded areas indicate 95% confidence intervals.