Democracy-at-Risk: Estimate Vulnerability and Resilience of Democracy

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Abstract

Political science is rich in theories, hypotheses, and data to examine political relationships that involve *non-linear* relationships between outcomes and explanatory variables. While qualitative studies have empirically tested these asymmetric relationships, quantitative empirical tools to adequately address them are limited. To address asymmetric hypotheses, this article proposes a method that estimates the conditional distribution, rather than just the mean, of the outcome variable as a function of explanatory variables using quantile regression and skewed t-distribution. By focusing on the conditional distribution, we can analyze the effects across the lower and upper tails of the probability distribution, capturing asymmetry through the empirical shape of the full distribution. I demonstrate the utility of this approach by exploring the risks of democratic vulnerability and resilience and their association with two economic drivers: income levels and income distribution. I find that when income level increases, the upside and downside democracy risks are symmetric, and regime stability increases with higher income across all levels of democracy. However, the effect of income inequality is asymmetric: while rising income inequality moderately increases the risk of democratic regression, it significantly decreases the probability of increasing the level of democracy.

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Introduction

A wide range of political science theories, hypotheses, and data examines the political relationship that is not *linearly* constructed. However, the standard empirical approach faces methodological challenges if the quantities of interest drawn from theories are not the conditional mean of the outcome variable on the explanatory variables. Many scholars have identified the issue of the mean-centric approach and proposed methods to estimate quantities of interests that do not rely on a *linear* assumption but allow for flexibly *asymmetry* assumption (Braumoeller and Goertz 2000; Clark et al. 2006; Goertz et al. 2012; Braumoeller 2006; Rosenberg et al. 2017).

Some scholars, for instance, argue that hypotheses about necessary conditions are effective in theorizing politics (Clark et al. 2006; Braumoeller and Goertz 2000). Clark et al. (2006) illustrate the neorealist war theory that claims anarchy causes war as one example. The theory implies that anarchy is a necessary, but not sufficient, condition for interstate war because we observe many cases of "not war" under conditions of anarchy. Other studies underscore the cases of data-generating processes that create boundary lines, either ceilings, floors, or both, that restrict the range of the observations and create truncated or censored data. Duverger's law is a good example of theories with a ceiling boundary. Duverger (1954) argues that single-member district plurality electoral systems are sufficient to produce a two-party system. This theory leads to the hypothesis that there is an upper limit on the effective number of parties in districts of a certain magnitude, suggesting no observation in the data where the effective number of seat-winning parties is larger than the district magnitude (Duverger 1954; Taagepera and Shugart 1993; Rosenberg et al. 2017).

To address the asymmetric hypothesis, scholars have proposed several methods. Multiplicative interaction term models are commonly used to test asymmetric hypotheses in quantitative research (Clark et al. 2006; Braumoeller and Goertz 2000). However, the approach relies on the deterministic assumptions of the relationship among variables, requiring modeling these assumptions prior to estimations, which is analogous to qualitative analysis,

such as Qualitative Comparative Analysis. Rosenberg et al. (2017) also point out the issue of the standard ordinary least squares (OLS) regression in multiplicative interaction term models because it ignores the data generation process that creates asymmetry due to the focus on estimating the central tendency of the data, the mean value of the outcome variable conditional on explanatory variables. To relax the deterministic assumption on asymmetry, more flexible approaches that estimate the degree of asymmetry from data are proposed. The methods include stochastic frontier analysis (Rosenberg et al. 2017), nonparametric frontier models (Rosenberg et al. 2017), and quantile regression models (Goertz et al. 2012; Rosenberg et al. 2017). These methods, however, have received little attention among political scientists, despite their potential to address important questions in political science.

In this article, I reevaluate the use of quantile regression to explore asymmetric hypotheses. I propose to estimate a conditional distribution, not just the mean, of the outcome variable as a function of explanatory variables with quantile regression. Following the method employed by Adrian et al. (2019), I map the estimated quantile coefficients into the skewed t-distribution, a flexible distribution function that indicates the estimated conditional distribution of the outcome variable. Whereas in standard linear models, a conditional mean, or expected value, is calculated as a single value by multiplying each of the estimated effects by the probability of each effect occurring and then summing all of those values, estimating the conditional distribution disaggregates effects at the lower and upper tails of the probability distribution, and thus captures asymmetricity through allowing either the upper or lower tail to be fatter or thinner.

To illustrate the theoretical and methodological significance of this approach, I estimate the resilience and vulnerability of democracy conditional on economic factors. In this application, I focus on two primary economic factors which have long been examined by political economy literature on democratization and consolidation: levels of income and income inequality. For each of the two factors, the literature has distinguished between theories of democratization that take the probability of a transition to democracy as their dependent

variable and theories of consolidation that take the probability of a democratic breakdown as their dependent variable. In particular, Przeworski and Limongi (1997) distinguish the former as an "endogenous" theory and the latter as an "exogenous" theory in the relationship to the levels of income, and then they reject only the endogenous modernization theory, showing asymmetricity between the two theories (Przeworski and Limongi 1997; Przeworski et al. 2000). Likewise, the theories on the association between income distribution and democracy do not only rely on a linear assumption. Whereas Boix (2003) argues the linear relationship between income distribution and democracy, claiming that decreasing the level of economic inequality bolsters democratization, Houle (2009) theorizes the asymmetry of the democracy-inequality relationship: inequality decreases democratic consolidation but has no net effect on democratization.

Using quantile regression, I estimate the conditional distribution of democracy as a function of economic factors to quantify the risks of democratic vulnerability and resilience. The conditional distribution provides three components that determine the shape of the distribution: 1) the mean of the distribution, which provides an expected value of the level of democracy analogous to the conditional mean of standard linear models, 2) the variance of the distribution, and 3) the skewness of the two tails of conditional distribution that captures asymmetry. To quantify the asymmetric risks from the conditional distribution, I estimate the probability of changing the level of democracy at a certain level. I call such probabilities as Democracy-at-Risk (DaR), dividing into the downside Democracy-at-Risk estimating from the lower tail of the distribution, and the upside Democracy-at-Risk from the upper tail. Democracy-at-Risk represents not only the mean effect of explanatory variables but also integrates the uncertainty through variance and the asymmetry through skewness of both tails of the distribution.

Democracy-at-Risk is also a flexible measure of the risk of regime stability and vulnerability. If you are interested in the democratization of authoritarian regimes, the upside Democracy-at-Risk is the quantity of interest, calculating the probability of the level of

democracy rising above a certain level[1]. Researchers also allow to estimate the risks of democratic consolidation and regression, by comparing the upside and downside Democracy-at-Risk. Theoretically, in the terminology of Democracy-at-Risk, Przeworski et al. (2000) predict the asymmetric effects of income level on the level of democracy, expecting that the downside Democracy-at-Risk is smaller (less likely to be autocratic) while the upside Democracy-at-Risk is larger (more likely to be democratic).

To demonstrate the effects of the level of income and income equality, I construct counterfactual scenarios in which I change each economic factor while holding everything else constant. In the scenarios of changing the level of income, I find a symmetric shift of the upside and downside Democracy-at-Risk. The estimated distribution shows that both lower and upper tails become parallelly thinner when GDP per capita increases, suggesting that levels of income contribute to regime stability at any level of democracy, either democracy or autocracy. This result is contradicted by the asymmetric assumption in Przeworski's theory.

On the other hand, the effect of income inequality is asymmetric. When income inequality increase, the probability of rising the level of democracy (the upside Democracy-at-Risk) decreases, while the downside Democracy-at-Risk increases. In other words, income inequality in democracy asymmetrically impacts more on democratic backsliding and less on democratic consolidation.

The contributions of this article are both methodological and substantive. First, I demonstrate how researchers can explore the asymmetric assumption by using quantile regression to answer important questions in political science fields. In recent years, quantile regressions have methodologically developed in the statistical and econometric literature to apply longitudinal data and binary, categorical data. This evolution has attracted scholars who study economic and financial data, where there are asymmetric data-generating processes. For instance, the IMF and BIS have used the conditional distribution approach to forecast

[1] You can use the continuous measures of democracy to set the cutoff value to distinguish democracy and autocracy. For instance, Freedom House defines 2.5 or below (from 1 to 7 scales), Polity defines 6 or above (from -10 to 10 scales), and V-Dem defines 0.5 or above (from 0 to 1) as the cutoff to democracy.

economic growth rates and inflation (Adrian et al. 2019, 2018; Banerjee et al. 2020). While some political scientists conduct quantile regression (Alexander et al. 2008, 2011; You et al. 2015; Goertz et al. 2012; Rosenberg et al. 2017), this paper is, to my best knowledge, the first to employ this method to political institutions to quantify the risks around the determinants of changes.

Second, this article expands on existing scholarship on democratic backsliding, the incremental erosion of democratic institutions, rules, and norms that stem from fair and competitive elections. A large number of scholars have studied the causes of democratic backsliding, including political polarization (Haggard and Kaufman 2021), the rise of populism (Berman and Snegovaya 2019; Prato and Wolton 2018), and the decline in foreign support for democracy (Hyde 2020). Revisiting both the modernization theories (Lipset 1959) and the distributive conflict theories (Boix 2003; Acemoglu and Robinson 2006), this article draws an integrated picture of both theories. The empirical results contribute to showing the nuanced implications of the cause of democratic backsliding.

Advantages of Quantile Regression

Studies of political regimes use the democracy measure as a continuous value for examining a nuanced and subtle change in the quality of democracy, such as democratic backsliding. The continuous democracy indexes show a bimodal distribution that skews to the higher score, a cluster of consolidated democracies, and the lower score of consolidated autocracies (Treisman 2020). Figure 1 plots the liberal democracy index (`v2x libdem`)[2] provided by V-Dem Institute (Coppedge et al. 2022) against the log of GDP per capita for 119 countries from 1980 to 2017, and the density plot on the right of the scatter plot indicates the bimodal distribution of the liberal democracy index.

The OLS regression techniques provide summary point estimates that calculate the average effect of the set of explanatory variables on the *average country in democracy index space*.

[2]I multiply the liberal democracy index by 100 for convenience.

However, this focus on the *average country* may hide important features of the underlying relationship when we are interested in the relationship at different points in a continuous democracy measure. For instance, in Figure 1, I draw the slope coefficients of the quantile regression of the liberal democracy index associated with the 5th (red line) and 95th (blue line) quantile, and the OLS estimates (black line). The red 5th quantile slope is flatter than the OLS estimates, as following consolidated authoritarian countries such as China (purple diamond). On the other hand, the blue 95th quantile slope captures some consolidated democracies such as Sweden (pink asterisk) and the United States (blue triangle). Thus, the OLS may not appropriately represent the changes from upper quantiles to upper quantiles or from lower quantiles to lower quantiles. Quantile regression techniques can therefore help us obtain a more complete picture of the underlying relationship between democracy and a set of economic and political factors.

Another advantage of quantile regression is that it avoids the restrictive assumption of normally distributed errors, and thus the approach is robust to outliers and heavy-tailed distributions. Since a political regime is sometimes changed drastically due to coups or wars, the data includes outliers and skewed distribution within and across countries. Two highlighted countries in Figure 1 indicate such moves. As for the case of democratization, Bulgaria (red circle) shifts to the 95th quantile from below the 5th quantile, whereas Hungry (green square) experiences democratic backsliding, moving to below 50 of the liberal democracy index from the 95th quantile. Although Figure 1 only presents the 5th and the 95th quantile, by adding more quintiles, quantile regression allows us to estimate these heavy-tailed distributions together with the median. Thus, quantile regression is a powerful tool that makes the task of modeling distributions flexible without having prior assumptions on distributions, even when the underlying story is complex and multi-dimensional (Angrist and Pischke 2008).

Despite the advantage of quantile regression, the application of this method in the political science field has been limited. One reason is the difficulty in interpreting quantile

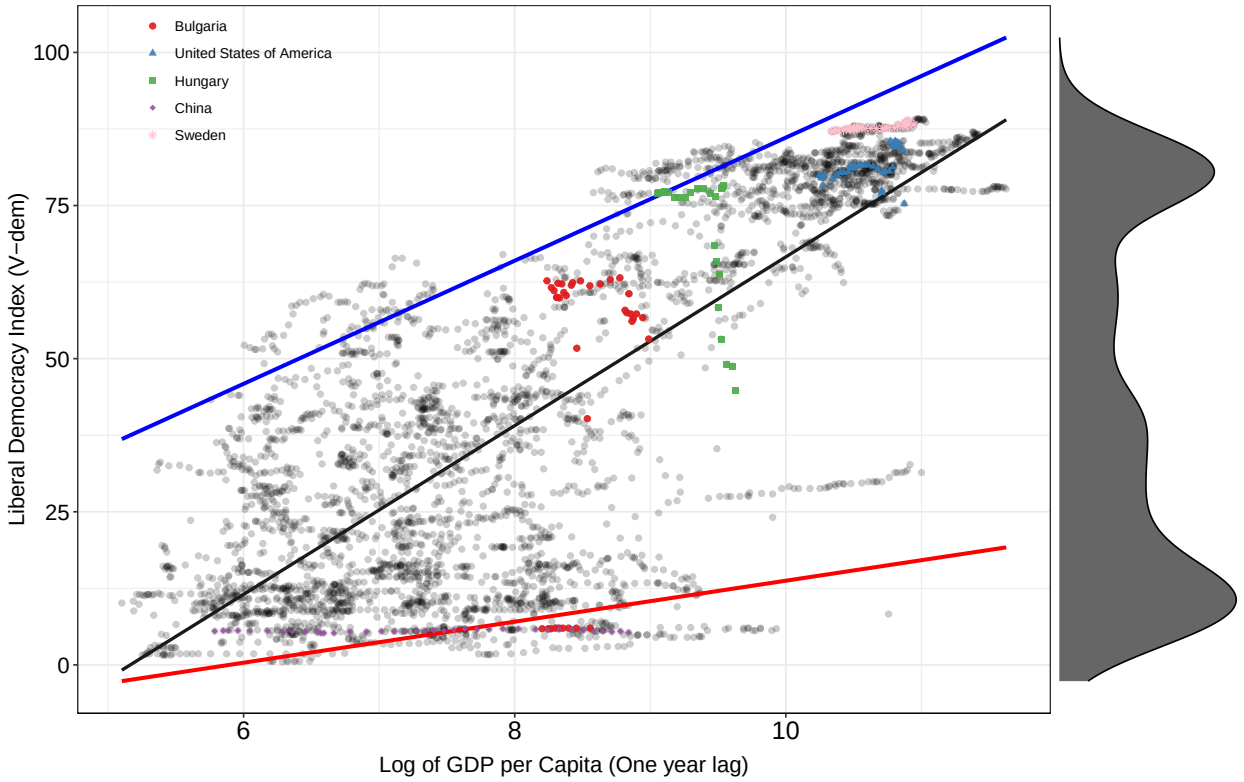

Figure 1: **Democracy Index and GDP per Capita with Quantile Regression Slopes.** Note: The data includes the liberal democracy index and GDP per capita for 119 countries from 1980 to 2017. The red, blue, and black lines are the slope coefficients of the quantile regression of the liberal democracy index associated with the 5th and 95th quantiles, and the OLS estimates.

regression results. While many studies present a set of coefficients at quantiles in a regression table or a line plot of their coefficients, these results can not intuitively capture the substantive meaning of the regression results. Meanwhile, Goertz et al. (2012) and Rosenberg et al. (2017) underscore the advantage of the use of quantile regression to test asymmetric hypotheses by identifying a quantile that creates an asymmetric relationship between the outcome variable and explanatory variables. But Rosenberg et al. (2017) admit the limitation that this approach can only be applied to bivariate settings. They also cast doubt on the substantive implications from the varied quantiles, and thus they argue that researchers are constrained to use the 5th or 95th quantiles as a fixed obvious choice (Goertz et al. 2012; Rosenberg et al. 2017).

However, recent developments in quantile regression methods provide tools for better es-

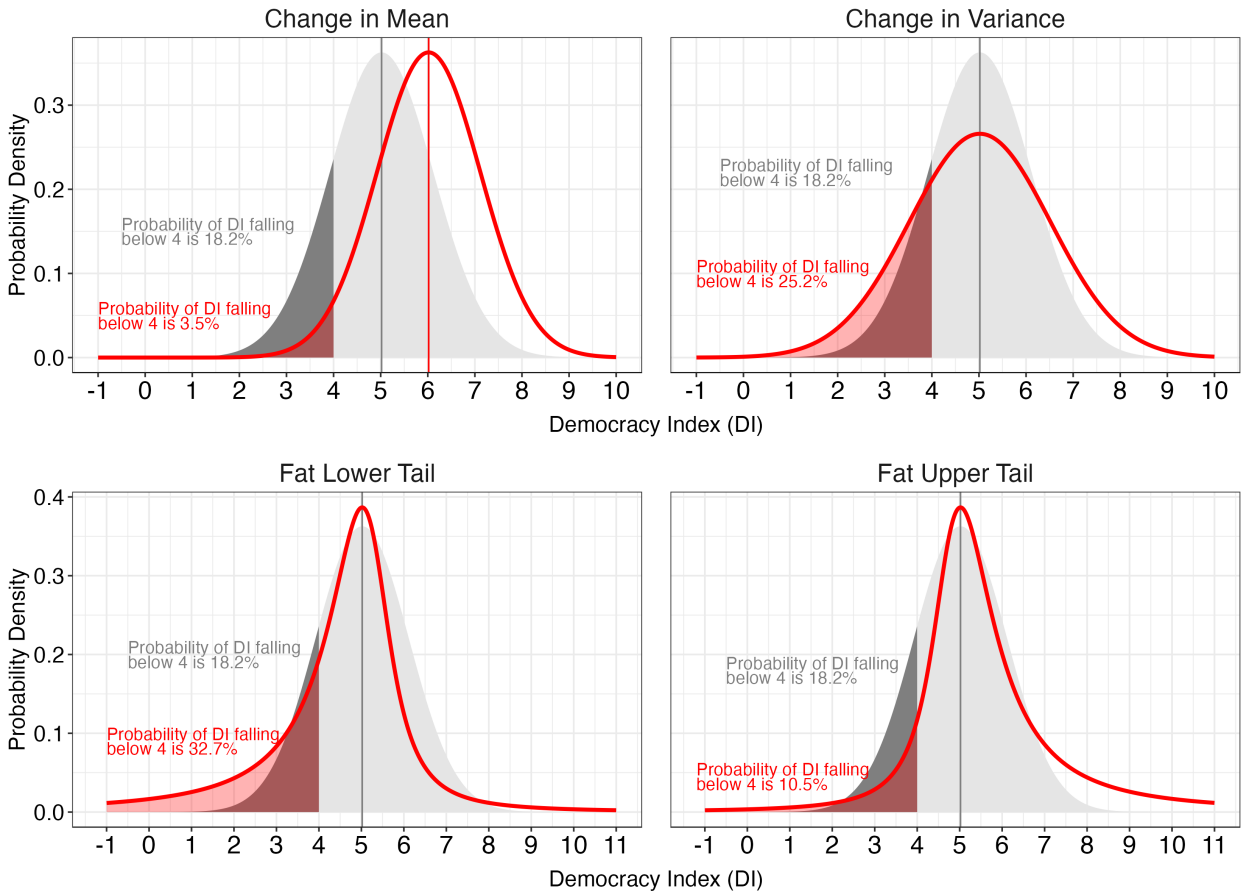

Figure 2: **Probability of Changing Democracy Index with Conditional Distribution Variation.**
Note: The gray shaded areas represent the baseline distribution, while the red solid lines represent the conditional distribution with changes in mean, variance, and upper and lower tails. The darker gray and red shaded areas indicate the probability of the democracy index falling below 4.

timation and interpretation. As discussed in detail later, Adrian et al. (2019) fit the quantile regression coefficients into the skewed t-distribution in order to smooth the quantile function and indicate a probability density function, allowing us to understand the probability distribution given the set of explanatory variables. The conditional distribution provides quantities of interest through the mean, variance, and skewness of the distribution.

The mean of the conditional distribution is analogous to the conditional mean function of standard regression models. As Figure 2 shows, the shift of the mean of the conditional distribution changes the probability of your quantity of interest in both tails. The top-left panel of Figure 2 indicates that the change of mean to 6 from 5 lowers the probability of the

democracy index falling below 4 to 3.5% from 18.2%. Likewise, the top-right panel indicates the change in the variance of the distribution. The narrower spread makes both tails thinner, suggesting the stability of the liberal democracy index increases. On the other hand, the wider spread increases the tail risks, making the state of regime vulnerable. The example in the top-right panel shows the wider spread case: the change of variance increases the probability of the democracy index falling below 4 to 25.2% from 18.2%. Lastly, skewness informs the asymmetric effect of explanatory variables on the democracy index. The bottom-left and bottom-right panels of Figure 2 indicate that the distributions are skewed, making the lower tail thin in the bottom-left and fat in the bottom-right, while the upper tail becomes thinner in the bottom-left and fatter in the bottom-right panel. Thus, this change of shape decreases the probability of the democracy index falling below 4 to 10.5% from 10% in the bottom-left panel and increases the probability to 32.7% from 18.2% in the the bottom-right panel.

Leveraging these methodological developments in the quantile regression approach, I empirically estimate the conditional distributions to analyze the determinants of nuanced changes in the level of democracy. In the next section, I will illustrate that asymmetric assumptions embedded in the theories of democratic transition and consolidation can fit into this proposed method.

Asymmetric Assumptions in Theories of Democracy

The statistical relationship between democracy and economic factors has been a center of debate in comparative politics. Since Lipset developed modernization theory (Lipset 1959), scholarship has examined the causal effect of economic factors on democratization. To capture more nuanced changes in the level of democracy, recent studies of democratic backsliding also examine these theories of democratic transitions and breakdowns (Waldner and Lust 2018). In this section, revisiting the debates on democracy and economic factors, I demon-

strate how existing theories can be translated into theories about asymmetric relationships which can be tested using quantile regression.

Modernization Theory

Scholarship on the relationship between income levels and democratization finds evidence for both democratic transitions and consolidation. Lipset (1959) argues that there is a positive relationship between the level of economic development and democratization, hypothesizing that as societies develop economically, the emergence of the middle class enables them to challenge a dictatorship. Economic development can bring about social changes, such as the spread of education, the acceleration of communication, and the expansion of mobilization and incorporation, that help regimes transition to democracies (Huntington 1968). Thus, the original modernization theory simply posits that the levels of income affect democratization or increase the democracy index, while the theory does not indicate the direction of autocracy.

On the other hand, first Przeworski and later Acemoglu and his colleagues argue against modernization theory, assuming the asymmetric relationship between economic development and democracy. Przeworski et al. (2000) dismiss the one direction of the relationship, which economic development increases the risk of democratization, because countries may randomly democratize due to reasons unrelated to their level of economic development (dismiss endogenous modernization theory). Conversely, they support another direction of the relationship that economic development decreases the risk of autocratization because once democracies experience higher economic development, they are less likely to slip into autocracy (exogenous modernization theory). Thus, this school of studies posits that levels of income asymmetrically impact the political regime transition.

The left panel of Figure 3 demonstrates to map Przeworski's idea into the shape of the conditional distribution. Because the theory argues increasing the level of economic development lower the risk of falling into authoritarian rule, the lower tail of conditional distribution indicates a thin tail. On the other hand, the theory predicts democratization is

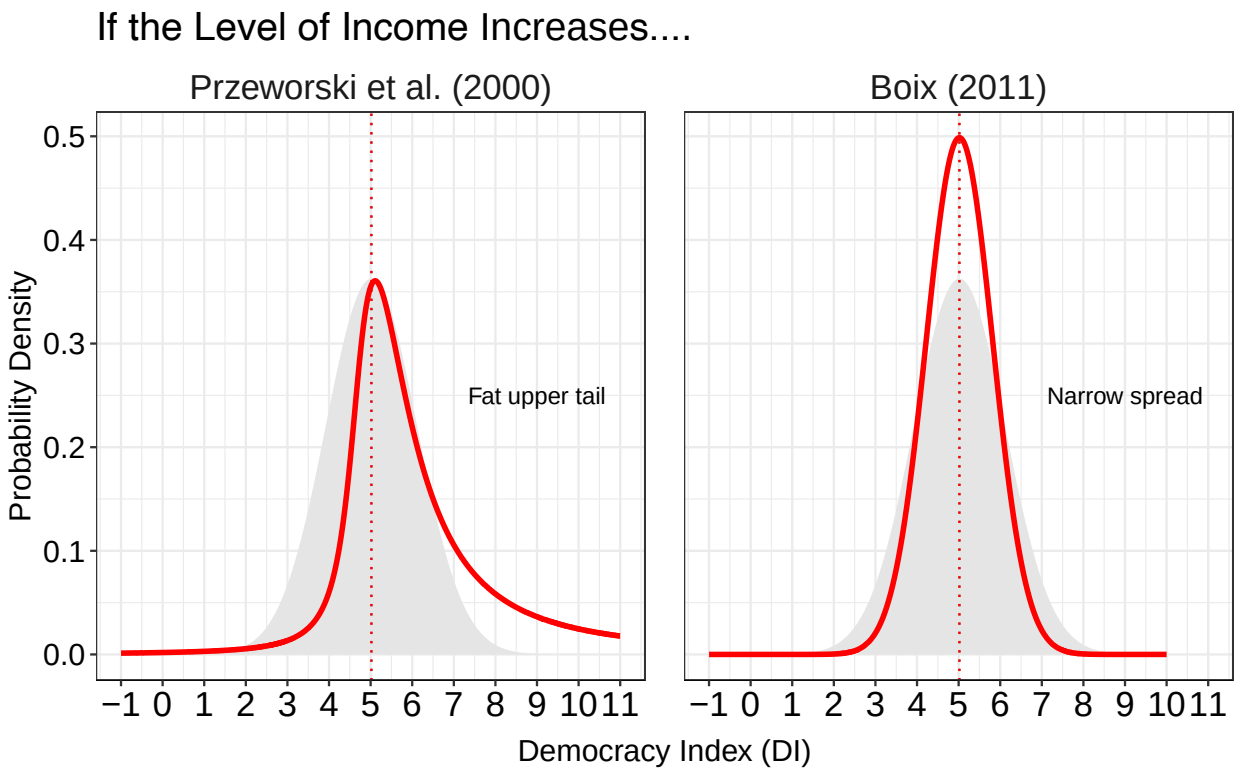

Figure 3: **Predicted Change in Democracy based on Theories of Democracy and Income level.**
Note: The gray shaded areas represent the baseline distribution, while the red solid lines represent the predicted conditional distribution with increases in the level of income based on the theories by Przeworski et al. (2000) and Boix (2011). The dotted lines indicate the mean value of each conditional distribution.

random, meaning the uncertainty or variance is large. Thus, the upper tail of conditional distribution shows a fat tail. Therefore, the upside Democarty-at-Risk (DaR) is larger, whereas the downside Democracy-at-Risk is smaller when the level of economic development increases.

In contrast, Boix (2011) develops his argument by adding conditions in his response to Acemoglu's criticism. He demonstrates temporally heterogeneous effects of endogenous modernization, which appear to be strong for the 19th century, moderate for the interwar period, and barely distinguishable from zero for the postwar period because the effect of economic development is strongly mediated by the structure of the international system (Boix 2011; Bermeo and Yashar 2016). Boix (2011) also argues that wealthy countries do not increase

the likelihood of democratization with any extra growth, instead extra growth stabilizes the state of the political regime. Thus, Boix (2011) suggests that economic development has little impact on the level of democracy.

As the previous chapter of the dissertation reveals, my argument is closely tied to Boix (2011) because I expect that levels of income have little effect in the sample of countries during and after the third wave of democracy. Due to the dynamic change in the international order created after WWII, this period includes not only the spread of democracy to a wide range of developing and former Soviet countries but also several cases in which countries regressed to an authoritarian regime (Boix 2011; Haggard and Kaufman 2012). At the same time, we observe a rise of institutionalized authoritarianism in which a variety of institutions in dictatorships help their regime survive (Gandhi and Lust-Okar 2009; Svolik 2012; Wright 2008). These institutions, including elections, pastries, legislatures, and executive constraints have lessened the risk of democratic transition compared to the period before the third wave.

I also illustrate Boix's idea in the shape of the conditional distribution in the right panel of Figure 3. Because Boix's argument underscores the stability of their regime rather than changing the level of democracy, he predicts that increasing the level of economic development lowers the variance of the distribution. Thus, the downside and upside Democracy-at-Risk will be smaller as both tails become thin. But, as the right panel of Figure 3 shows, Boix expects that the mean of the distribution does not change, predicting that the economic development does not change the mean of the conditional distribution, but variance.

Distributive Conflict Theory

When it comes to income inequality, the seminal work of Meltzer and Richard (1981) provides the original idea behind the distributive conflict models of regime change. The Meltzer-Richard model posits the conflict between the wealthy minority and the poor majority. In such a case, because politicians appeal to the median voter, the wider the divergence between the median and mean income, the more one may expect generous tax and transfer programs

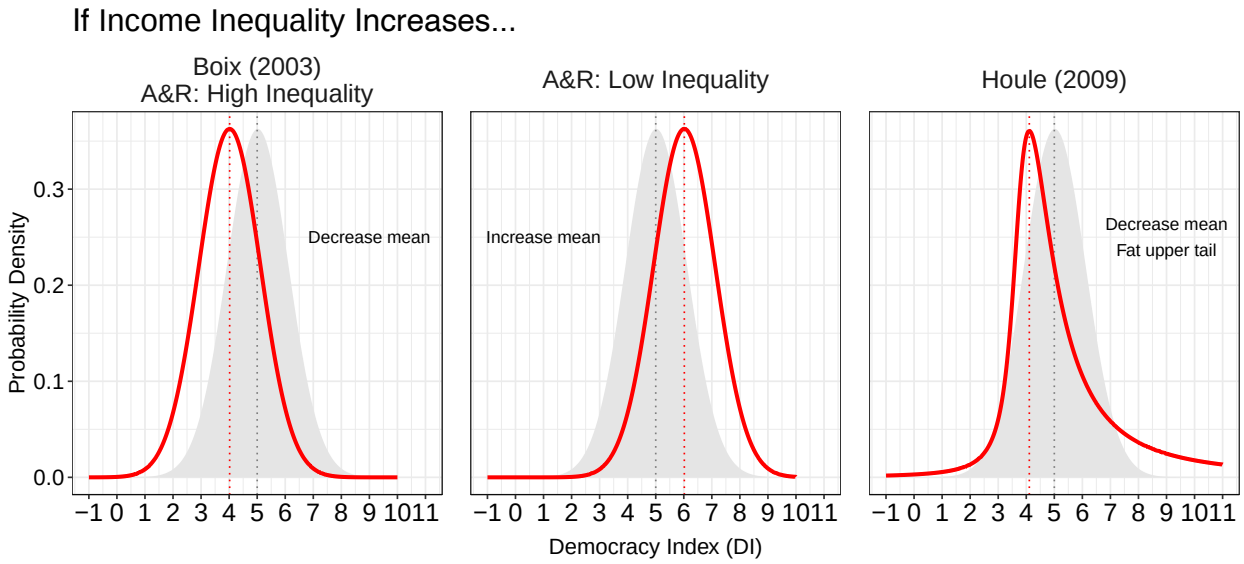

Figure 4: **Predicted Change in Democracy based on Theories of Democracy and Income Inequality.**
Note: The gray shaded areas represent the baseline distribution, while the red solid lines represent the predicted conditional distribution with increases in income inequality based on the theories by Boix (2003), Acemoglu and Robinson (2006), AR, and Houle (2009). The dotted lines indicate the mean value of each conditional distribution.

that redistribute resources from the wealthy to the poor.

Boix (2003) and Acemoglu and Robinson (2006) build on the Meltzer-Richard model to extend the model of transitions and consolidation of democracy, assuming strategic interactions between elites and masses. Boix (2003) argues that the effect of inequality is linearly negative: democratization is less likely when income inequality increases. Based on the assumption that the decision-making power rests with elites and not with masses, Boix (2003) argues that elites are more likely to grant democracy when inequality is low because the costs of repression outweigh those of being taxed. As the left panel of Figure 4 shows, this liner assumption can simply be put into conditional distribution analogous to the OLS, because it only changes the mean of the conditional distribution.

Likewise, Acemoglu and Robinson (2006) also argue that the relationship between inequality and democracy is symmetric but has an inverse-U shape, assuming two economic conditions. On the one hand, in the lower income inequality condition, the probability of democratization linearly increases as income inequality increases until it gets to the middle

levels of inequality where elites have less to fear from redistribution, but the masses still have incentives to democratize. In this hypothesis, democratization is unlikely to occur in authoritarian governments with low levels of inequality because the demand of the poor for redistribution is also attenuated. On the other hand, in the higher income inequality condition, the probability of democratization declines as income inequality increases. The latter assumption is similar to Boix's argument that elites repress the masses when inequality becomes significant. Thus, this idea can be illustrated in the same conditional distribution in the left panel of Figure 4. Conversely, the center panel indicates the former scenario under the lower income inequality, expecting that the mean of the conditional distribution increases when income inequality increases.

Other scholars, however, propose an alternative hypothesis based on a linear assumption. Houle (2009) argues that the impact of inequality on elites and masses is ambiguous if democratization requires elites to acquiesce to the demands of the poor. Haggard and Kaufman (2012) also supports this view and highlights that theoretical models proposed by Acemoglu and Robinson (2006) depends on various parameters, such as the cost of repression or asset mobility, due to potential indeterminacy. Thus, these scholars expect that increasing income inequality has no net effect on democratization. Rather, these scholars argue that increasing income inequality promotes democratic breakdowns because redistributive pressures from the poor motivate elites to deploy force against incumbents in order to reimpose authoritarian rule (Houle 2009). These hypotheses expect that the mean of the conditional distribution decreases as income inequality rises. It also predicts the asymmetric tails because increasing income inequality has no effect on the rise of the level of democracy, expecting the variance of the upper tail is small, and thus, the tail is thin. On the other hand, the lower tail of the conditional distribution is fat because the theory argues increasing income inequality increases the risk of falling into authoritarian rule, indicated in the left panel of Figure. 4.

Estimation Strategy

Measuring Quantile Effects

To estimate the conditional relationship between democracy, income level, and income inequality, I conduct quantile regressions with fixed effects. Let us denote by $Y_{t+1,i}$ the democracy index of country i at time $t+1$. I use the liberal democracy index (`v2x libdem`) provided by V-Dem Institute (Coppedge et al. 2022) for Y[3]. I consider the following model:

$$Y_{i,t+1} = \alpha_i + \beta X_{i,t} + U_{i,t} \tag{1}$$

where α_i is a fixed effect. The vector $X_{i,t}$ contains three variables: the lag of the dependent variable ($Y_{i,t}$), the log of `GDP per capita` and `Income Inequality Ratio`. The data for `GDP per capita` is from World Bank and Penn World Table. `Income Inequality Ratio` is calculated by the ratio of the average incomes of the top 10% to the bottom half, using World Inequality Database.

Following the model proposed by Koenker (2004), the conditional quantiles for the democracy index are obtained as

$$Q_{Y_{i,t+1}}(\tau \mid X_{i,t}) = \alpha_i + X'_{i,t}\beta(\tau) \tag{2}$$

By definition, the τ-th quantile of the distribution of democracy is the value $Q_Y(\tau)$. For instance, when $\tau = .10$, $Q_{Y_{i,t+1}}(\tau \mid X_{i,t})$ describes the lower decile of $Y_{i,t+1}$ given $X_{i,t}$, while $\tau = .5$ gives us the conditional median. The coefficient α_i captures time-invariant country-specific effects that may shift the averaged location of distribution for each country i. More specifically, the fixed effects do not capture the country-specific association among quantile coefficients (shape), but it does capture the parallel shift of quantile coefficients (location) to calculate the average coefficients across countries.

In a quantile regression of $Y_{i,t+1}$ on $X_{i,t}$, the regression slope β_τ is chosen to minimize

[3] I multiply the liberal democracy index by 100 for convenience to avoid the long decimal points.

the quantile weighted absolute value of errors, whereas ordinary least squares regressions minimize the value of squared errors. Koenker (2004) proposes a method that treats unobservable fixed effects as parameters to be jointly estimated with the covariate effects for different quantiles[4]. Specifically, parameter estimates are estimated as follows:

$$\begin{aligned}&\left(\hat{\beta}\left(\tau_k,\lambda\right),\{\alpha_i(\lambda)\}_{i=1}^{N}\right)\\&=\arg\min\sum_{k=1}^{K}\sum_{t=1}^{T}\sum_{i=1}^{N}w_k\rho_{\tau_k}\left(Y_{i,t+1}-\alpha_i-x_{i,t}'\beta\left(\tau_k\right)\right)+\lambda\sum_{i=1}^{N}\left|\alpha_i\right|\end{aligned}\tag{3}$$

where $\rho_\tau = u(\tau = I(u < 0))$ denotes the piecewise linear quantile loss function ($I(.)$ is an indicator function) proposed by Koenker and Bassett (1978)[5], and w_k is the relative weight given to the k-th quantile, which controls for the contribution of the k-th quantile on the estimation of the fixed effects. In this paper, following the previous studies (Alexander et al. 2011; You et al. 2015; Lamarche 2010), I employ equally weighted quantiles $w_k = 1/K$. λ is the tuning parameter that shrinks the individual effects toward zero to improve the performance of the estimate of β. When the term λ goes to zero, the penalty term disappears, and we obtain the usual fixed effects estimator; when the term λ goes to infinity, we obtain the estimate of the model without the individual effects (Pooled model). I set $\lambda = 1$, following You et al. (2015).

I estimate coefficients for nine quantiles: 10, 20, 30, 40, 50, 60, 70, 80, and 90 percent quantiles. The confidence intervals are computed by block bootstrapping using country clusters with 1,000 replications.

[4] This methodological development is important. One of the technical issues why quantile regression is not widely used among political scientists is that, while quantile regression can conduct interaction terms or instrumental variables as a conventional regression does, the application of these methods to panel data is methodologically limited. Because political science research frequently employs time-series data and its techniques, researchers need to employ proper tools, such as fixed effects or instrumental variables, to identify causal effects (under suitable assumptions).

[5] See also Angrist and Pischke (2008).

Fitting t-distribution to Estimate Conditional Distribution

To substantively understand the set of quantile coefficients, I fit the result of quantile regressions to the skewed t-distribution in order to smooth the quantile function and estimate conditional distribution. Theoretically, quantile regression provides us with approximate estimates of the quantile function, an inverse cumulative distribution function of the democracy index. In practice, these estimates are difficult to map into a probability distribution function because of approximation error and estimation noise.

To solve this problem, following the approach conducted by Adrian et al. (2019), originally developed by Azzalini and Capitanio (2003), I fit the skewed t-distribution in order to smooth the quantile function and indicate a probability density function.

$$f(\hat{Y}|X,\mu,\sigma,\alpha,\nu) = \frac{2}{\sigma} t\left(\frac{\hat{Y}-\mu}{\sigma};\nu\right) T\left(\alpha\frac{\hat{Y}-\mu}{\sigma}\sqrt{\frac{\nu+1}{\nu+\left(\frac{\hat{Y}-\mu}{\sigma}\right)^2}};\nu+1\right) \tag{4}$$

where $t(\cdot)$ and $T(\cdot)$ respectively denote the PDF and CDF of the Student t -distribution.

The four parameters of the distribution pin down the location μ, scale σ, fatness ν, and shape (degree of freedom) α. Relative to the t-distribution, the skewed t-distribution adds the shape parameter, which regulates the skewing effect of the CDF on the PDF. I estimate the four parameters $\{\mu_{i,t}, \sigma_{i,t}, \alpha_{i,t}, \nu_{i,t}\}$ of the skewed t-distribution f to minimize the squared distance between the estimated quantile function $Q_{Y_{i,t+1}}(\tau \mid X_{i,t})$ from the equation (2) and the quantile function of the skewed t-distribution $F^{-1}(\tau;\mu_{i,t},\sigma_{i,t},\alpha_{i,t},\nu_{i,t})$ from the previous equation (4) to match the $10, 20, 30, 40, 50, 60, 70, 80$ and 90 percent quantiles:

$$\{\hat{\mu}_{i,t+1},\hat{\sigma}_{i,t+1},\hat{\alpha}_{i,t+1},\hat{\nu}_{i,t+1}\} = \underset{\mu,\sigma,\alpha,\nu}{\arg\min} \sum_{\tau} \left(\hat{Q}_{y_{i,t+1}|x_t}(\tau \mid x_{i,t}) - F^{-1}(\tau;\mu,\sigma,\alpha,\nu)\right)^2 \tag{5}$$

The approach can draw the conditional distribution for any given pair of year t and country i or hypothetical scenario to simulate the effect of economic factors.

Quantities of Interest from Conditional Distribution

To understand the substantive impact of the economic variables, I conduct two counterfactual experiments that explore how the levels of income and income inequality affect the conditional distribution of the democracy index.

First, I create the baseline distribution by interpolating the mean values of all variables into the estimated quantile coefficients and then map t-distribution to obtain the conditional distribution. Next, I change the value of each economic factor by one standard deviation. To compare the baseline and the counterfactual distributions, I also calculate the Democracy-at-Risk as the probability that the democracy index changes to a certain threshold. These risks are two-sided, with the upside Democracy-at-Risk estimating from the upper tail that indicates the "risk" of increasing the democracy index, and the downside Democracy-at-Risk from the lower tail, the risk of decreasing the democracy index. Democracy-at-Risk allows research to ask a substantive question of democratization. For example, if you are interested in the risk of autocratization from democracy, you can set the threshold value of 50 for distinguishing between democracy and authoritarianism and thus the downside Democracy-at-Risk can be estimated as the probability of the level of democracy falling below 50.

Formally, let denote Y_A^* as a pre-specified threshold, and the conditional upside Democracy-at-Risk, $\mathrm{P}^{Down}(\overline{Y}) \equiv \mathrm{Prob}(\overline{Y} < Y^*)$, is the probability mass below Y^* in the conditional density $f(\overline{Y}|\mu, \sigma, \alpha, \nu)$:

$$\mathrm{P}^{Down}(\overline{Y}) \equiv \int_{-\infty}^{Y^*} f(\overline{Y}|\mu, \sigma, \alpha, \nu) d\overline{Y} \tag{6}$$

where at the probability of the democracy index falling below Y^* is (100-τ) percent[6]. In other words, this expression defines the downside Democracy-at-Risk through the integral of the PDF over the democracy index support up to a specified threshold or, equivalently,

[6] Similarly, we can define the conditional upside Democracy-at-Risk, $\mathrm{P}^{Up}(\overline{Y}) \equiv \mathrm{Prob}(\overline{Y} > Y^*)$, as $\mathrm{P}^{Up}(\overline{Y}) \equiv \int_{Y^*}^{\infty} f(\overline{Y}|\mu, \sigma, \alpha, \nu) d\overline{Y}$, which is the probability of the democracy index rising above Y^*.

through the CDF.

Estimation Results

Estimated Quantile Regressions

I first estimate the quantile coefficients before fitting them into skewed t-distribution. Figure 5 shows the raw estimated coefficients from the quantile regressions at the 10, 20, 30, 40, 50, 60, 70, 80, and 90 percent quantiles in the blue line, together with the least squares estimate in the green line. Figure 5 highlights nonlinearities associated with both the levels of income (the log of GDP per capita) and income inequality (income ratio of the top 10 percent to the bottom half). In particular, the slope of income levels decreases from the lower to upper quantiles. Whereas the coefficient of the OLS estimation is positive but not statistically significant, the coefficients in the upper quantiles are positive and statistically significant, and those in the lower quantiles are negative and statistically significant. In the case of the middle quantiles, the slope is close to zero, and the confidence intervals overlap with zero. This indicates that the effect of the level of income on the level of democracy for the middle quantiles is uncertain. On the other hand, the upper and lower quantiles exhibit heterogeneous effects. However, without a benchmark or baseline effect for comparison, it remains unclear the extent of this heterogeneity and its implications for the level of democracy.

In contrast, income inequality shows a negative relationship, particularly in the middle quintiles, where the effect is statistically significant. This suggests that income inequality has a significant and negative impact on the level of democracy, unlike the level of income. Both the upper and lower quantiles also display negative coefficients, although the confidence intervals overlap with zero. The effect size is larger for the upper quantile compared to the lower quantile. Again, however, evaluating this asymmetry and its implications on democracy is challenging without a benchmark or baseline effect for comparison.

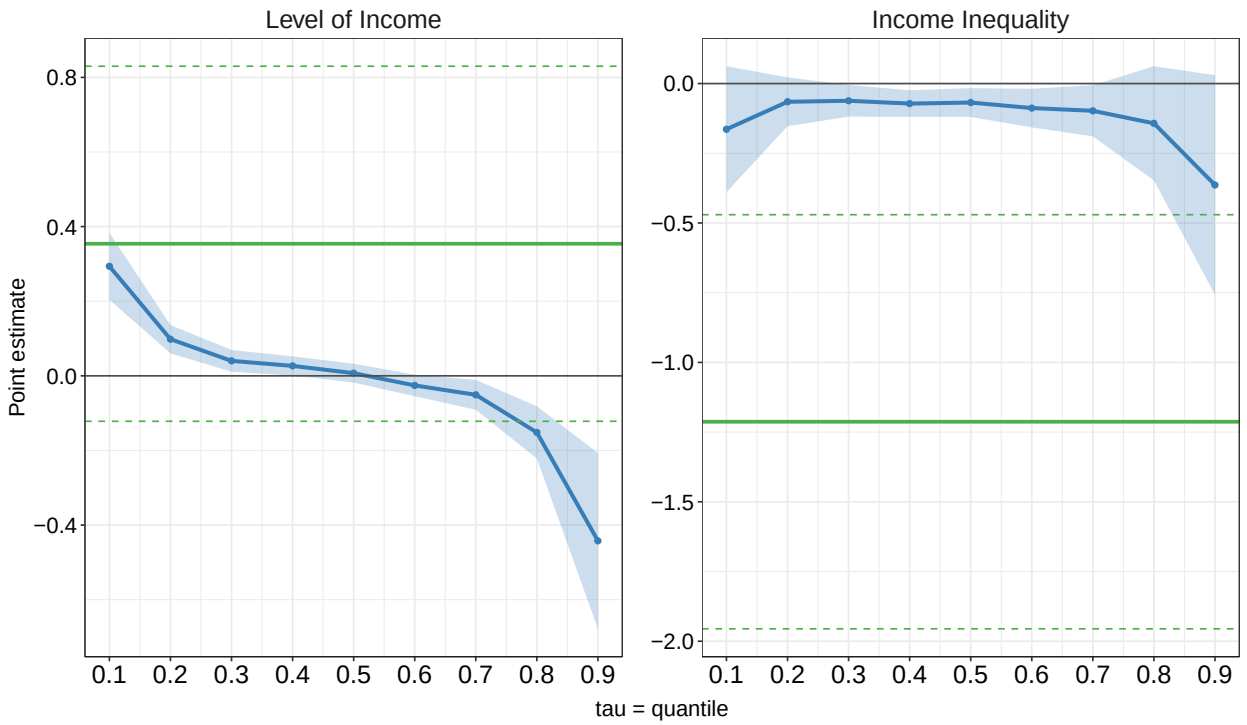

Figure 5: **Estimated Quantile and OLS Coefficients.**
Note: The blue shaded areas are the 95% confidence interval. The Coefficients of the OLS estimation and the 95% confidence intervals are indicated in the green solid and green dashed lines, respectively.

Estimated Conditional Distribution

To illustrate the effects of the changes in the political and economic factors on the conditional distribution, Figure 6 and Figure 7 indicate the conditional probability density functions for the democracy index, fitted as skewed t-distributions. As mentioned above, I set all variables to their means to estimate the baseline result shown as the shaded areas, and then create two hypothetical scenarios for each economic factor. First, I change the values of the level of income by one standard deviation holding other variables constant: the left panel of Figure 6 indicates the decrease of GDP per capita scenario, and the left panels of 6 indicate the increase of GDP per capita scenario. Next, I change the values of income inequality by one standard deviation holding other variables constant. Figure 7 shows this decrease and increase in income inequality scenarios.

To compute the Democracy-at-Risk, which represents the probability of changing the

level of democracy across various scenarios, I establish threshold values for both the upside and downside risks. This approach allows researchers to set the thresholds based on their specific interests flexibly. In this particular analysis, I focus on examining the marginal impact of each economic factor on the level of democracy for an *average country*. It is important to note that this *average country*, computed by taking the mean values of all variables, is purely hypothetical. As previously mentioned, the distribution of the level of democracy is bimodal, meaning that the mean value (42.0) does not accurately represent the majority of countries. Nonetheless, this hypothetical mean country serves as a useful baseline for setting the threshold and estimating the Democracy-at-Risk. In the baseline scenario, I calculate the level of democracy at the 10th and 90th quantile probabilities, as a 10% risk captures the marginal and asymmetric effects sufficiently. For the downside risk, the value is calculated to be 40.8, while it is 42.9 for the upside risk.

Figure 6 presents the scenario that changes the level of income, suggesting the symmetric impact. When it comes to the location of the mean of the distribution, neither scenario shows any change in their respective mean. Instead, as the level of income increases by one standard deviation, the variance of the distribution is squeezed. On the other hand, as the level of income declines, the variance of the distribution is stretched. The downside and upside Democracy-at-Risk are almost symmetrically changed in both scenarios. When the level of income decrease, the downside and upside Democracy-at-Risk increase to 18.8% and 18.4% from 10%. On the other hand, when the level of income increases, the downside and upside Democracy-at-Risk increase to 1.0% and 1.4% from 10%, suggesting the stability of the regime at any level of the democracy index increases.

The results of this study support Boix's theory while challenging Przeworski's asymmetric assumption. According to Przeworski et al. (2000), they predict that the probability of increasing the level of democracy would also increase as the level of income increases, resulting in a thicker upper-tail distribution. Similarly, they expect that as the level of income increases , the lower-tail distribution, representing the downside Democracy-at-Risk, should become

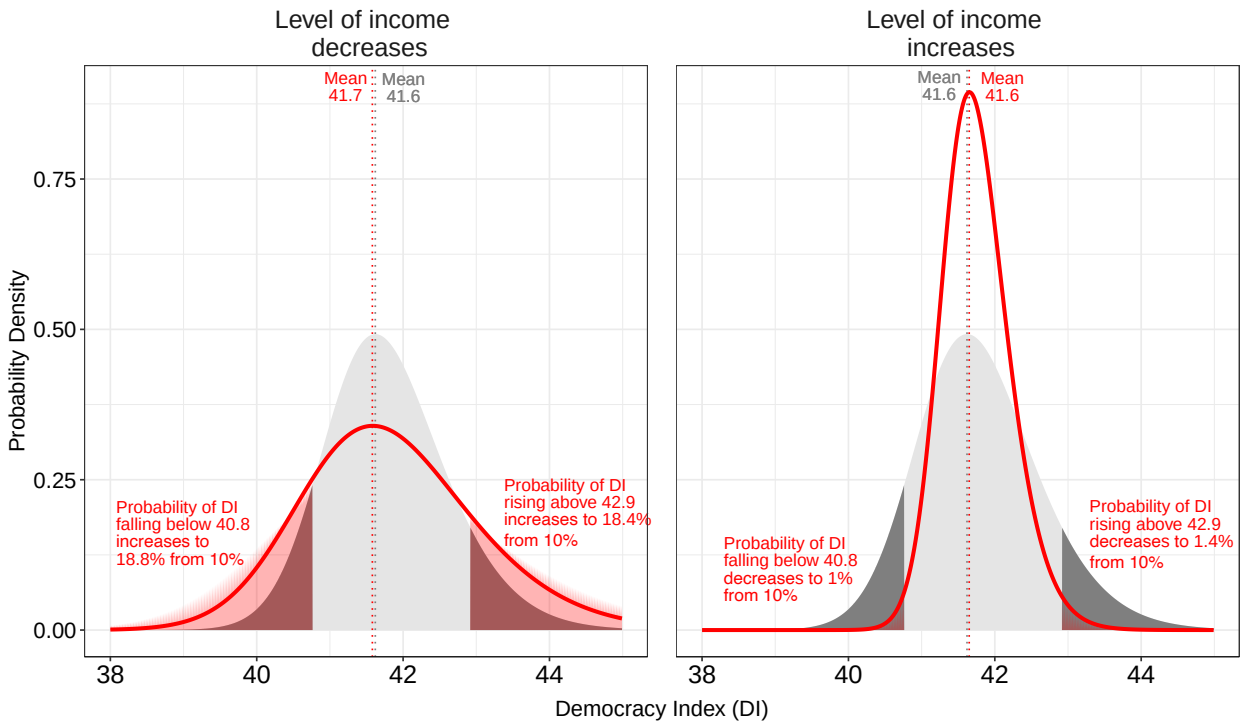

Figure 6: **Effect of Changes in the Level of Income on Probability Densities.** Note: The gray shaded areas are estimated as a baseline distribution by interpolating the mean values of all variables into the estimated quantile coefficients. The red solid lines indicate the estimated conditional distribution when the level of income changes one standard deviation from its mean. The darker gray shaded areas indicate the probability of the democracy index falling below 40.8 or rising above 42.9 at 10% confidence. The darker red areas also indicate the probability of the democracy index falling below 40.8 or rising above 42.9. The gray and red dotted lines are the mean of the distribution.

thinner. However, the results of this study indicate a symmetric shift in both tails of the distribution, contradicting Przeworski's prediction. Instead, as Boix (2011) expects, the level of income influences not the mean of democracy but variance. This suggests that both democracies and authoritarian countries are more likely to consolidate their political regimes because economic development may provide more resources to the incumbent to maintain their regimes. Consequently, significant regime shifts, such as democratization or democratic backsliding, become less likely due to the increase in the level of income.

In contrast, Figure 7 illustrates the asymmetric distributional changes. In the left panel, as the income inequality ratio decreases, the mean of the conditional distribution shifts slightly to the left. Conversely, in the right panel, an increase in income inequality is as-

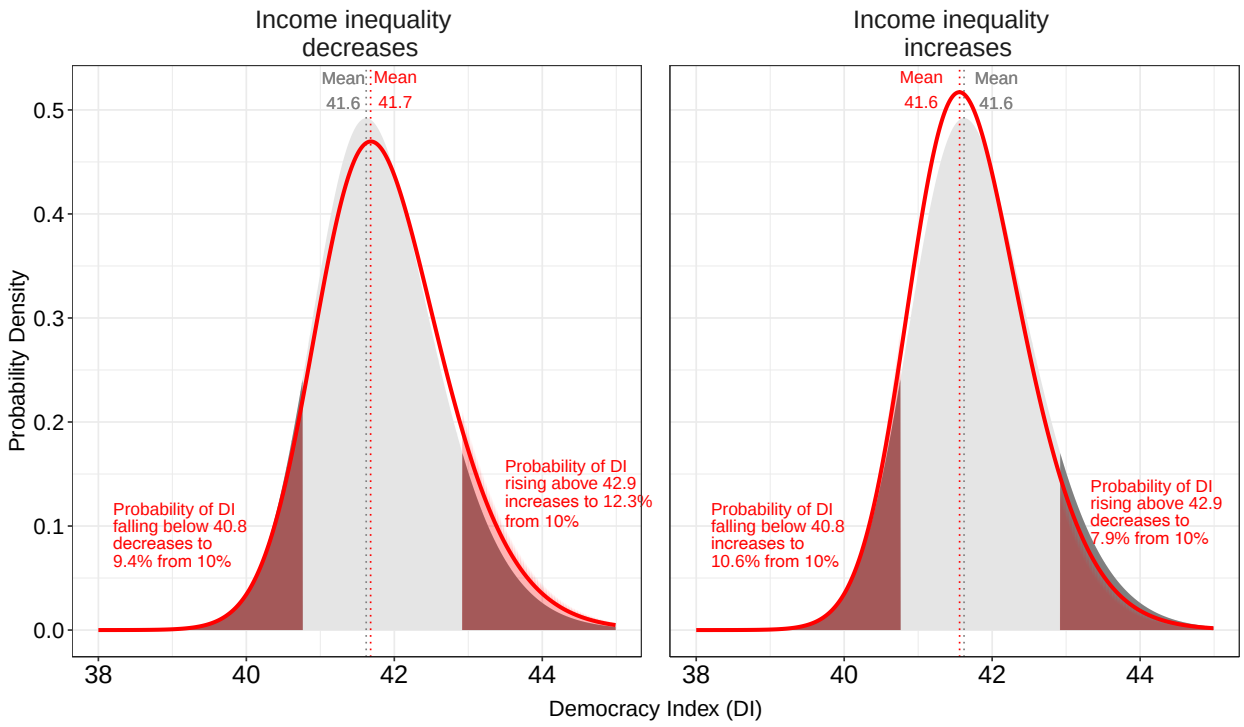

Figure 7: **Effect of Changes in Income Inequality on Probability Densities.**
Note: The gray shaded areas are estimated as a baseline distribution by interpolating the mean values of all variables into the estimated quantile coefficients. The red solid lines indicate the estimated conditional distribution when the level of income changes one standard deviation from its mean. The darker gray shaded areas indicate the probability of the democracy index falling below 40.8 or rising above 42.9 at 10% confidence. The darker red areas also indicate the probability of the democracy index falling below 40.8 or rising above 42.9. The gray and red dotted lines are the mean of the distribution.

sociated with slight leftward shifts. Additionally, the plot reveals that a decrease in the income inequality ratio leads to a fatter upper tail while the lower tail remains stable at the baseline distribution. Similarly, an increase in inequality results in a thinner upper tail, while the lower tail remains stable in the right panel of Figure 7. These changes in distribution asymmetry are also reflected in the downside and upside Democracy-at-Risk in both scenarios. When income inequality decreases, the downside Democracy-at-Risk decreases slightly from 10% to 9.4%, while the upside Democracy-at-Risk increases 12.3% from 10%. Conversely, when income inequality increases, the downside Democracy-at-Risk increases slightly to 10.6% from 10%, while the upside Democracy-at-Risk decreases 7.9% from 10%.

While Boix (2003) and Acemoglu and Robinson (2006) assume a linear relationship be-

tween democracy and income inequality, the shape of the estimated conditional distribution suggests an asymmetric relationship. The asymmetric assumption by Haggard and Kaufman (2012) and Houle (2009) is supported by the results, but the shape of asymmetry is the opposite. They argue that there is little association between income inequality and the upside Democracy-at-Risk, while the downside Democracy-at-Risk is larger because elites would impose authoritarian rule under redistributive pressures from the poor (Houle 2009). However, the estimated results present the risks are asymmetric in that the changes in the upside Democracy-at-Risk is larger than the downside Democracy-at-Risk.

It is beyond the scope of this article to explore the reasons behind this asymmetric relationship, but the results have implications for the analysis of democratic backsliding. Backsliding and consolidation entail nuanced changes in democratic rule rather than sudden transitions between different political regimes, as theorized by (Houle 2009). Therefore, the improvement of democratic governance is more likely to be influenced by the combined efforts of both the masses' social activities and the elites' endeavors. The assumption that democratization is exclusively determined by the equilibrium of strategic interaction between elites and masses may be overly restrictive when considering the gradual improvement of democratic rules. Consequently, an increase in income inequality may not necessarily lead to a significant alteration in the downside risk of democracy. This is because income inequality does not make the costs for elites of mobilizing against democratic rule outweigh the losses arising from redistribution under democratic rule.

Lastly, to compare the upside and downside Democracy-at-Risk for each economic factor, I conduct hypothetical scenario analyses where each economic factor takes values ranging from two standard deviations below the mean to two standard deviations above the mean, while other factors remain at their mean values. Similar to the previous approach, I set the threshold values for the upside and downside Democracy-at-Risk to be the democracy index when the probabilities in both tails are 10% in the baseline scenario[7]. Figure 8 illustrates

[7]Specifically, the values are 40.8 for downside risk and 42.9 for upside risk. I also conduct sensitivity analyses with different thresholds calculated through the probabilities in both tails are 5% and 25% in the

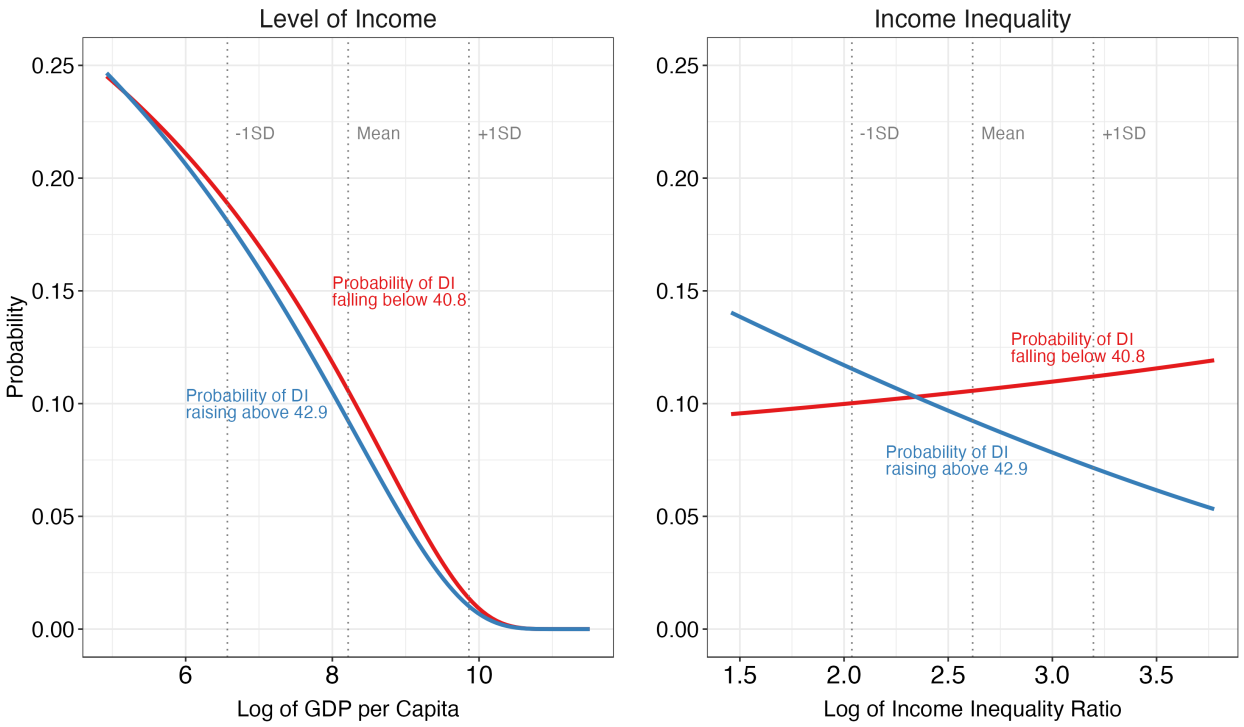

Figure 8: **Effect of Changes in Economic Factors on Democracy-at-Risk.**
Note: The red line indicates the downside Democracy-at-Risk, the probability of the democracy index falling below 40.8, and the blue line indicates the upside Democracy-at-Risk, the probability of the democracy index rising above 42.9. These thresholds are estimated from the probabilities in both tails are 10% in the baseline scenario. The gray dotted lines indicate the mean values of each economic variable, one standard deviation below and above the mean.

the downside Democracy-at-Risk using red lines and the upside Democracy-at-Risk using blue lines.

In the left panel of Figure 8, the downside and upside Democracy-at-Risk move in parallel. As the conditional distributions compress towards the mean, resulting in lower variances, the probability of the democracy index falling below 40.8 (or rising above 42.9) becomes zero as the level of income increases. This suggests that an increase in the level of income stabilizes the current state of the political regime at any level of the democracy index. On the contrary, as shown in the left panel of Figure 8, the downside and upside Democracy-at-Risk exhibit asymmetric movements. The downside Democracy-at-Risk, which represents the probability of the democracy index falling below 40.8, increases as the income inequality ratio rises,

baseline scenario. The details are described in Appendix

while the upside Democracy-at-Risk, representing the probability of the democracy index rising above 42.9, decreases. Furthermore, the slope of the upside Democracy-at-Risk is steeper compared to the slope of the downside Democracy-at-Risk. This suggests that the impact of income inequality on democratic consolidation is more significant than its impact on democratic regression. Therefore, similar to the previous analysis, these results highlight the symmetric effect of the level of income and the asymmetric impact of income inequality on the dynamics of democracy.

Conclusion

This article introduces the use of quantile regression and fitting the quantile coefficients to conditional distribution to explore asymmetric political relationships. Despite many political science studies embedding asymmetric assumptions in their theories, hypotheses, and data, these studies are mainly tested in qualitative approaches, such as Qualitative Comparative Analysis. Few scholars offer quantitative tools to explore asymmetric assumptions, and thus these tools are paid little attention. I propose the method to quantify asymmetry using quantile regression and the skewed t-distribution, demonstrating the effectiveness of the method in capturing nuanced impacts of economic factors on the level of democracy. While I focus on the theories on democracy in the application, this article provides a tool that allows researchers to examine a wide range of important questions in political science that have potentially asymmetric theories that have not been tested.

Nevertheless, the method proposed in this article has room for further development. First, it is difficult to use the shape of conditional distribution and the downside and upside risk of each factor in statistical decisions because the estimations of the skewed t-distribution do not provide confidence intervals. While the applications in this article, as well as previous studies (Adrian et al. 2019, 2018; Banerjee et al. 2020), treat the quantile coefficients as known at the first stage, future research could propagate the estimation uncertainties into

the skewed t-distribution. Second, the method can be extended to examine more complicated and nuanced theories and data by relaxing the assumption of the proposed method. For instance, the fixed effect quantile regression model in this article only captures the parallel shift of quantile coefficients (location) to calculate the average coefficients across units. But the recent studies of quantile regression offer the fixed effect model that estimates the unit-specific association among quantile coefficients that differentiate the skewness of conditional distribution (Machado and Silva 2019). Likewise, future studies could incorporate instrument variables to have a robust causal identification strategy, estimate the quantile coefficient for count and binary data, depending on data and application, or use the Bayesian approach to fitting the skewed t-distribution, instead of maximum likelihood, to explore the more complex data that require prior distribution to converge.

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Appendix

Sensetive Analysis for Effect of Changes in Economic Factors on Demoracy-at-Risk

In this analysis, following Figure 8, I conduct hypothetical scenario analyses where each economic factor takes values ranging from two standard deviations below the mean to two standard deviations above the mean, while other factors remain at their mean values. As for sensitive analysis, I set the threshold values for the upside and downside Democracy-at-Risk to be the democracy index when the probabilities in both tails are 5% and 25% in the baseline scenario. The specific values of the thresholds are summarized in the table below.

Table A1: Threshold Values for Upside and Downside Democracy-at-Risk

Probability	Upper Threshold	Lower Threshold
5%	43.3	40.5
10% (Fig 8)	42.9	40.8
25%	42.3	41.2

Similar to Figure 8, the results in Figure A1 present the symmetric effect of the level of income and the asymmetric impact of income inequality on the dynamics of democracy.

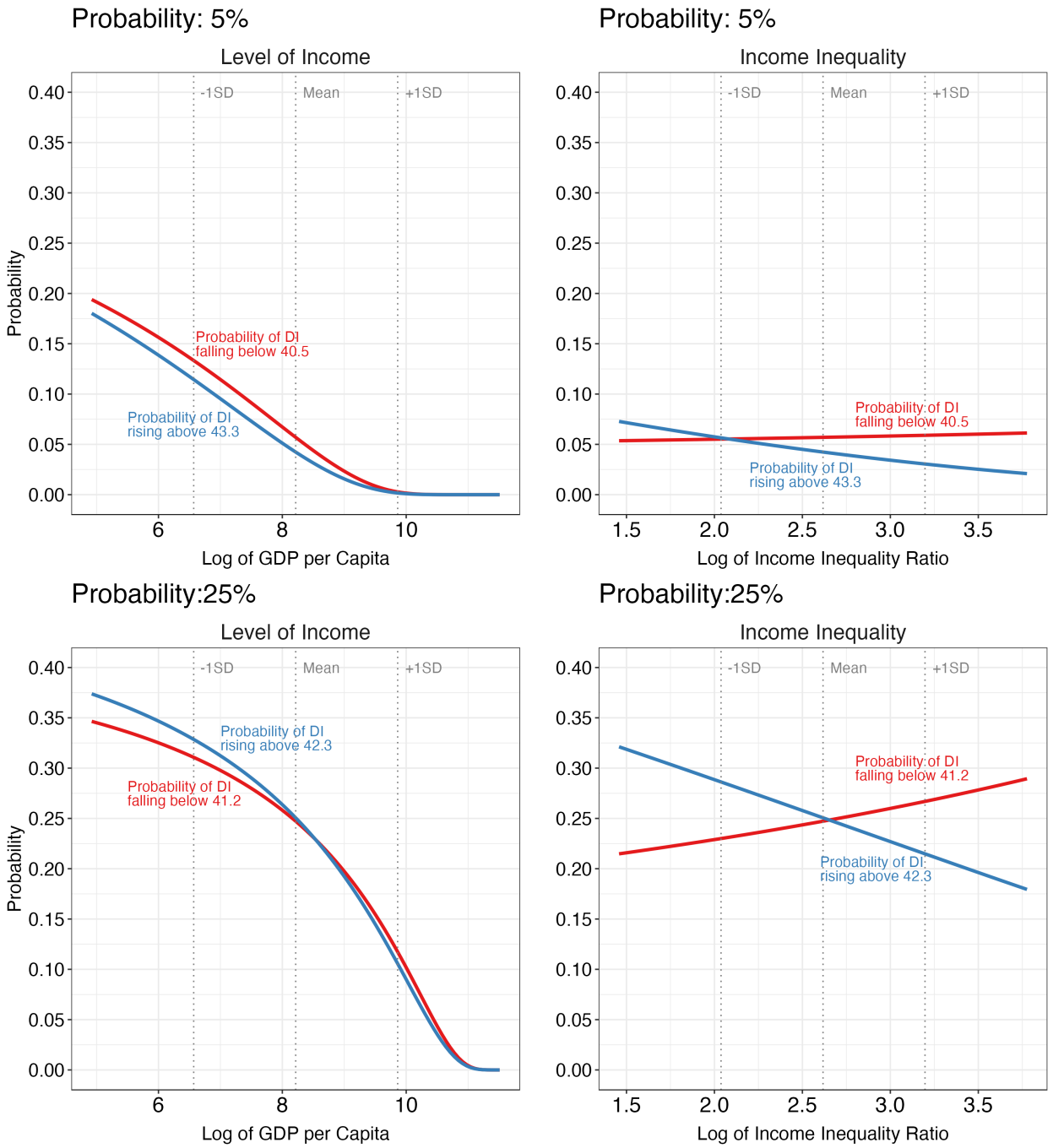

Figure A1: **Effect of Changes in Economic Factors on Democracy-at-Risk.**
Note: The red line indicates the downside Democracy-at-Risk, the probability of the democracy index falling below 40.5 for 5% scenario and 41.2 for 25% scenario, and the blue line indicates the upside Democracy-at-Risk, the probability of the democracy index rising above 43.3 for 5% scenario and 42.3 for 25% scenario. The gray dotted lines indicate the mean values of each economic variable, one standard deviation below and above the mean.